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Adaptive Shape Control for Aerodynamic Design

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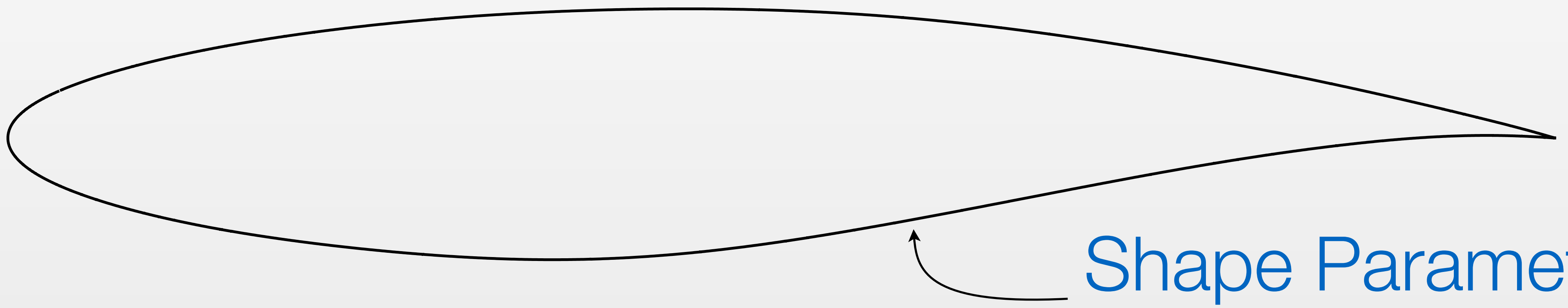
NASA ARMD 2013-2015 Seedling Fund effort

Parametric Shape Optimization

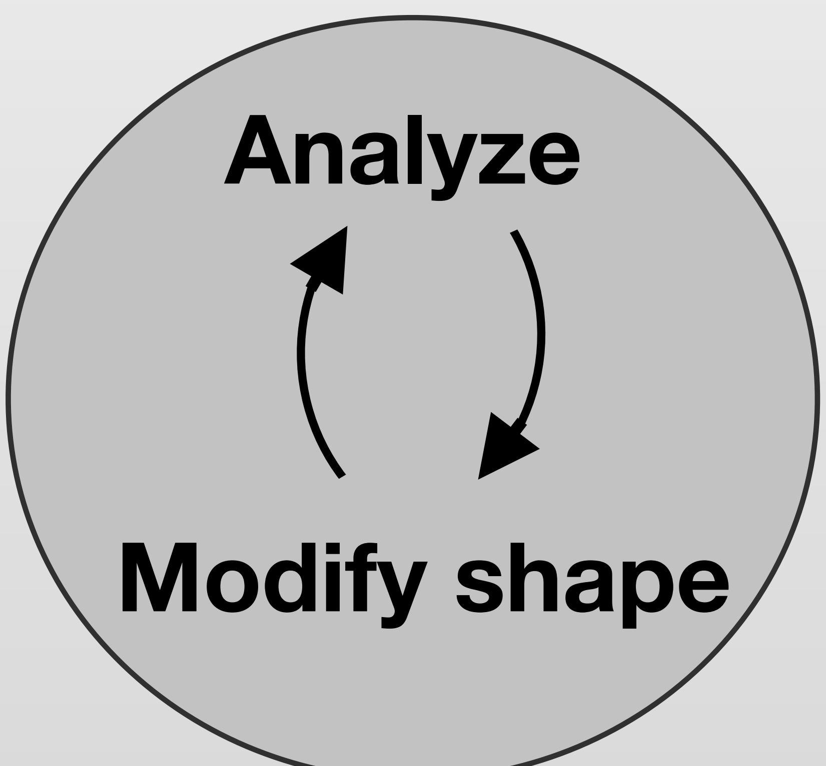
Start with baseline aerodynamic shape



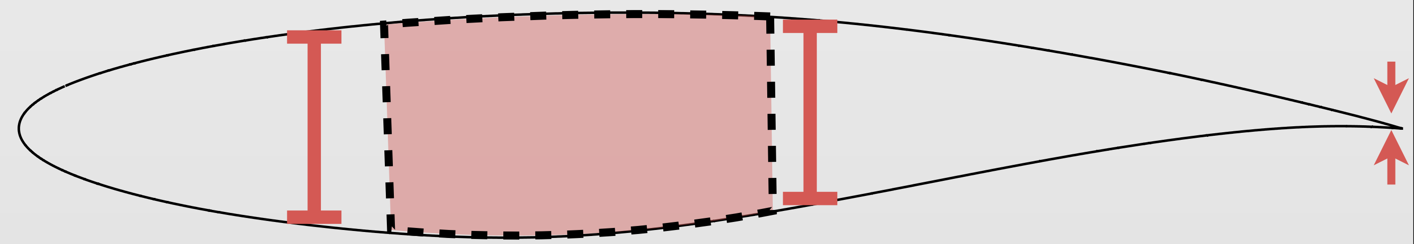
Shape Parameters

$f(x) = \mathbf{a_0}\, x^3 + \mathbf{a_1}\, x^2 + \mathbf{a_2}\, x + \mathbf{a_3}$

Iteratively modify shape
to improve performance



Constraints



Objective

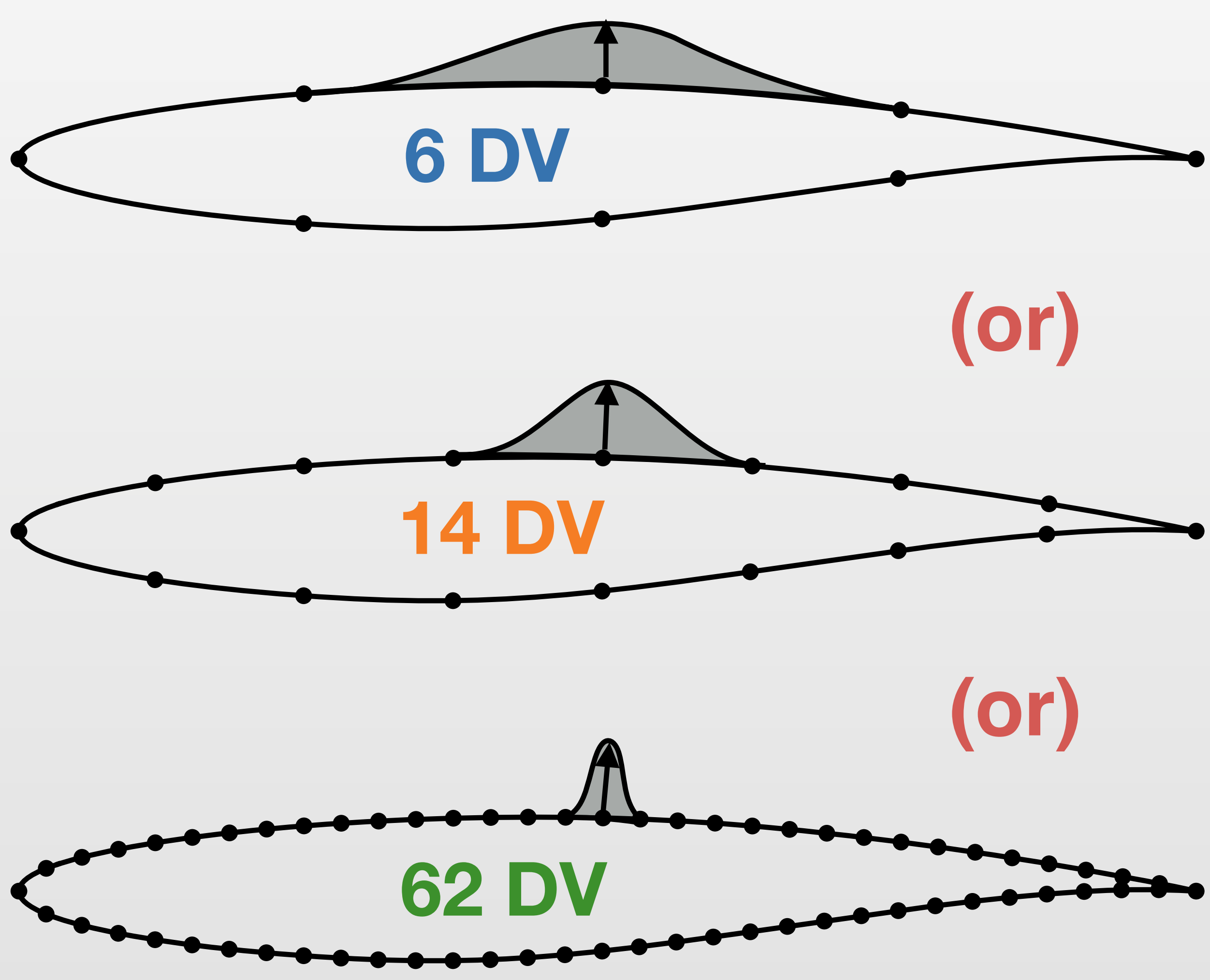
Flow variables

$$\min C_D(\mathbf{Q}(\mathcal{S}))$$

Surface

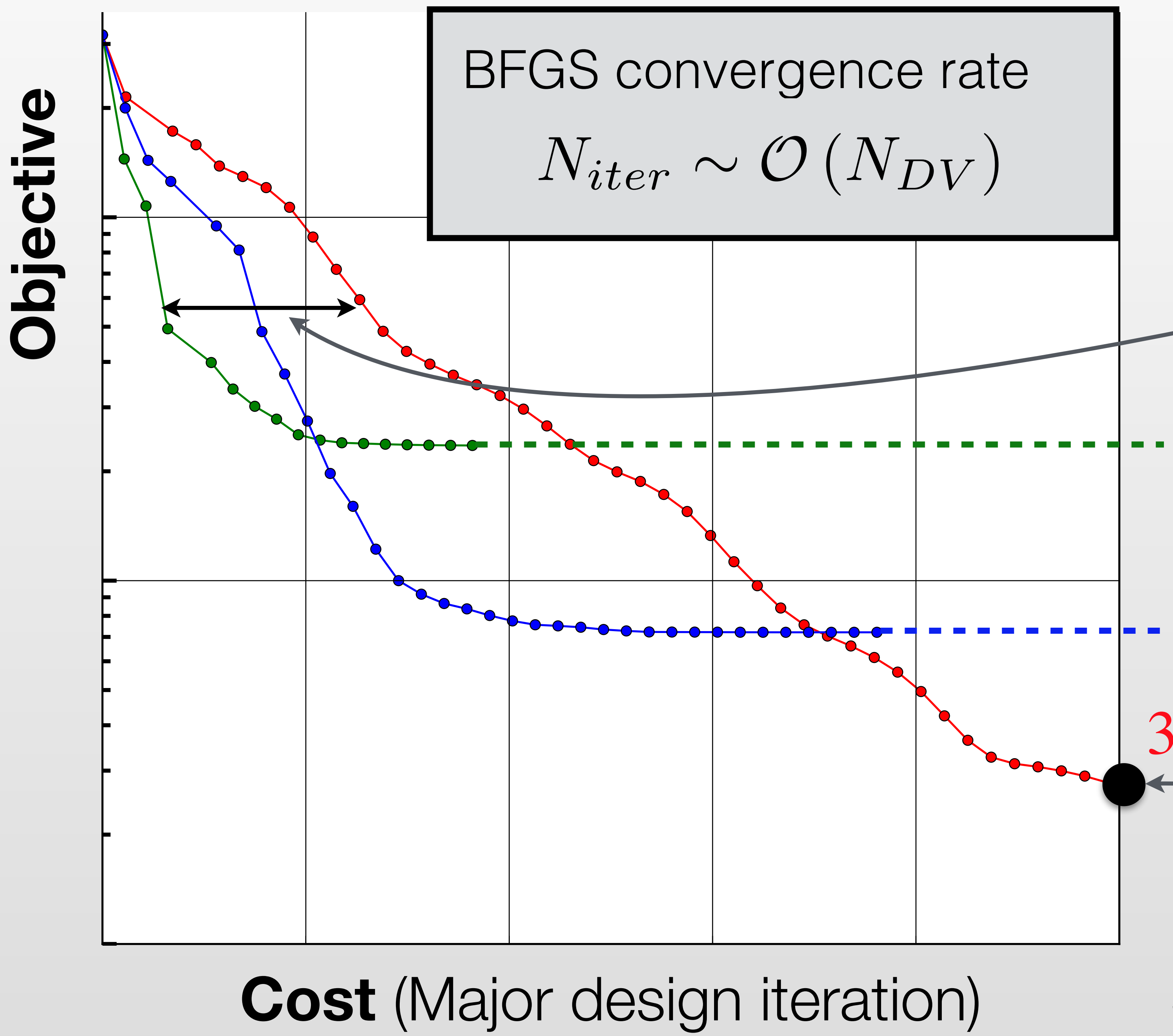
Motivation

Scalable Resolution



How many shape parameters are needed?

Motivation

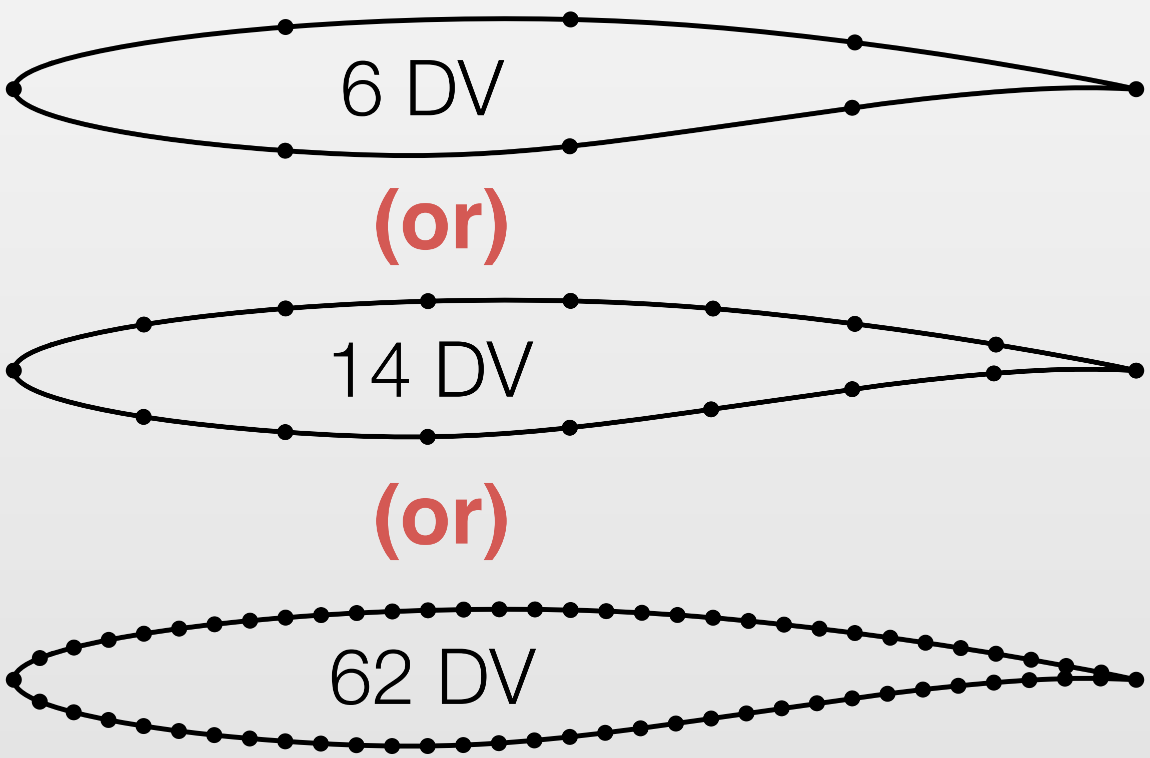


Low resolution:
Faster design improvement

High resolution
Superior final design

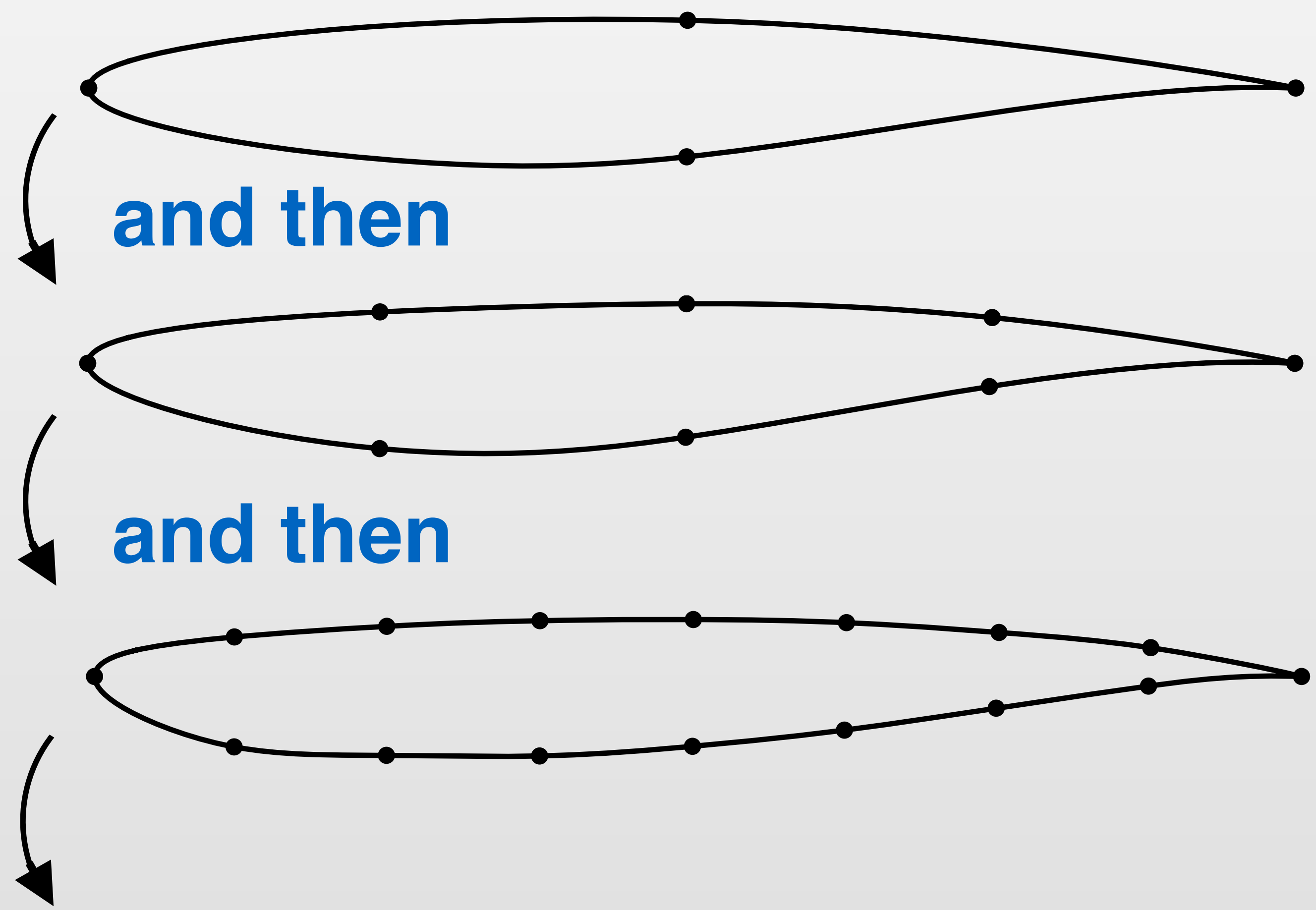
Progressive Parameterization

Instead of choosing a **static** (fixed) parameterization…



Optional **manual** refinement to continue

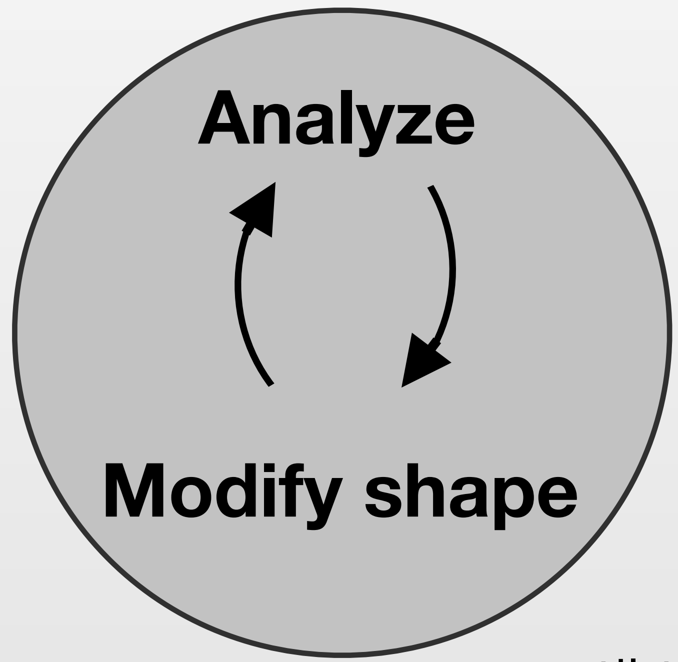
…**Progressively** refine the shape control concurrently with optimization.



etc…

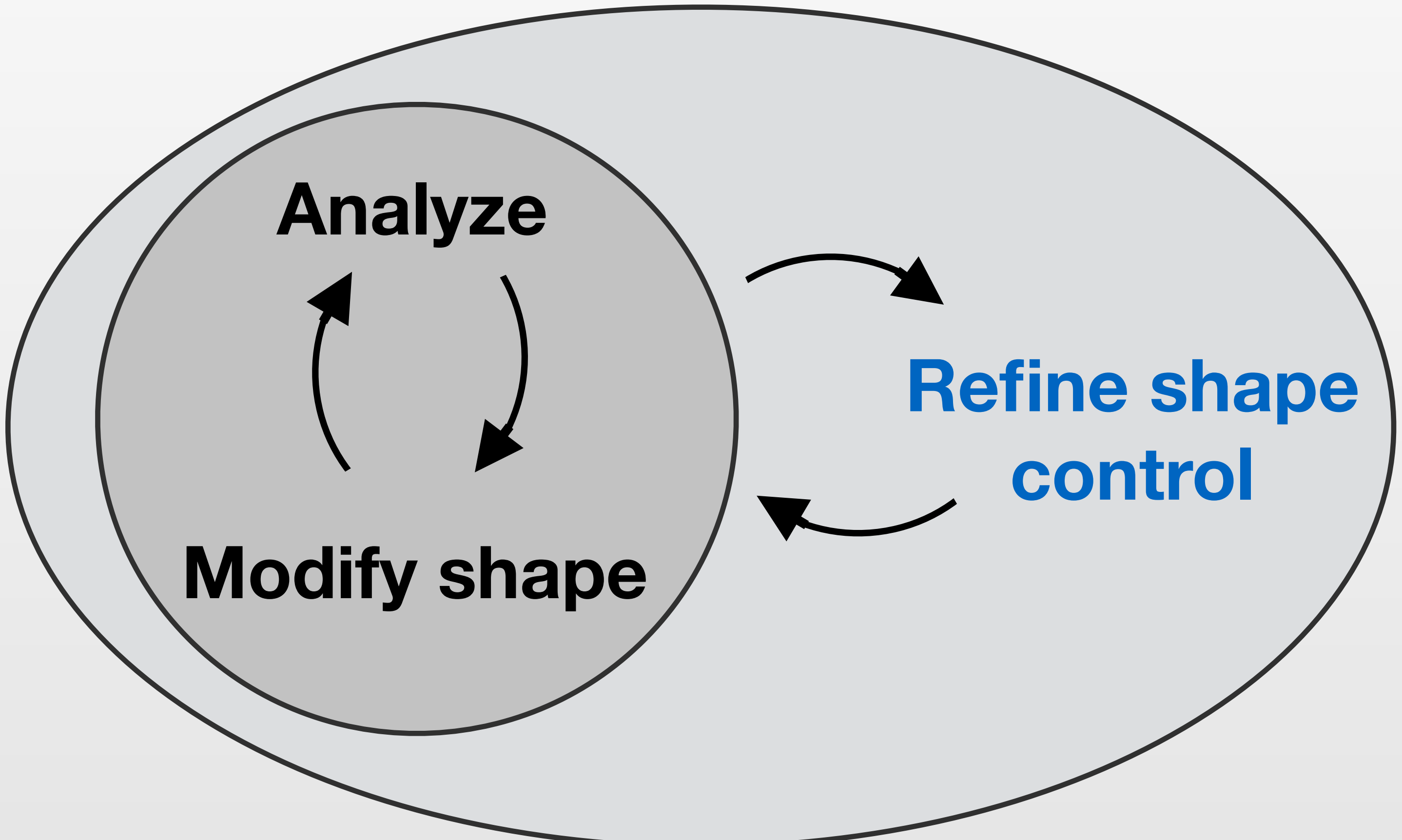
Automatic refinement

Optimization Loop



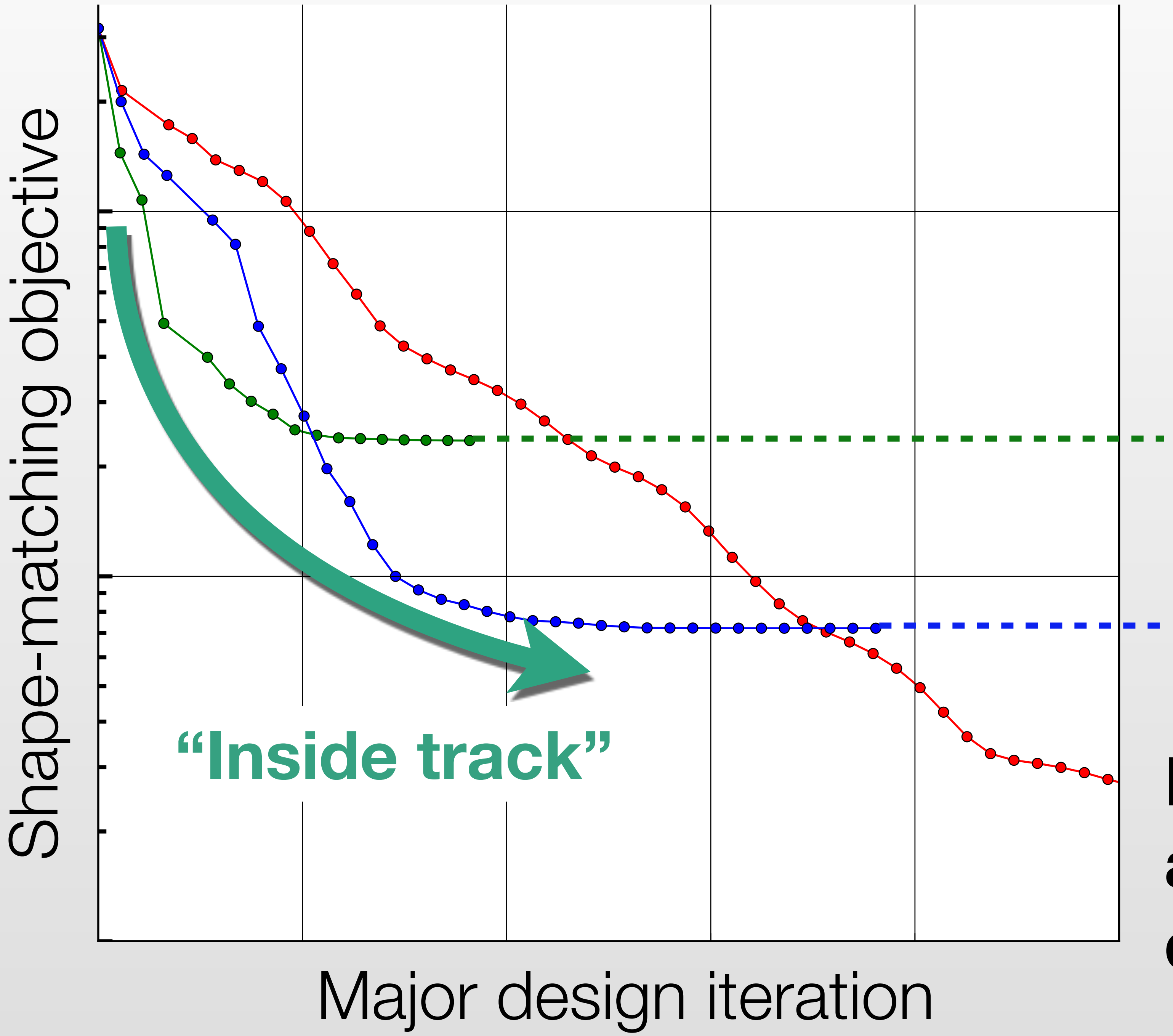
until the objective converges with respect to the shape **parameters**.

Optimization Loop



until the objective converges with respect to the shape control refinement.

Motivation



Introduce new degrees of freedom **only when necessary** to improve the design.

In the limit of refinement, we **approach the continuous design space**.

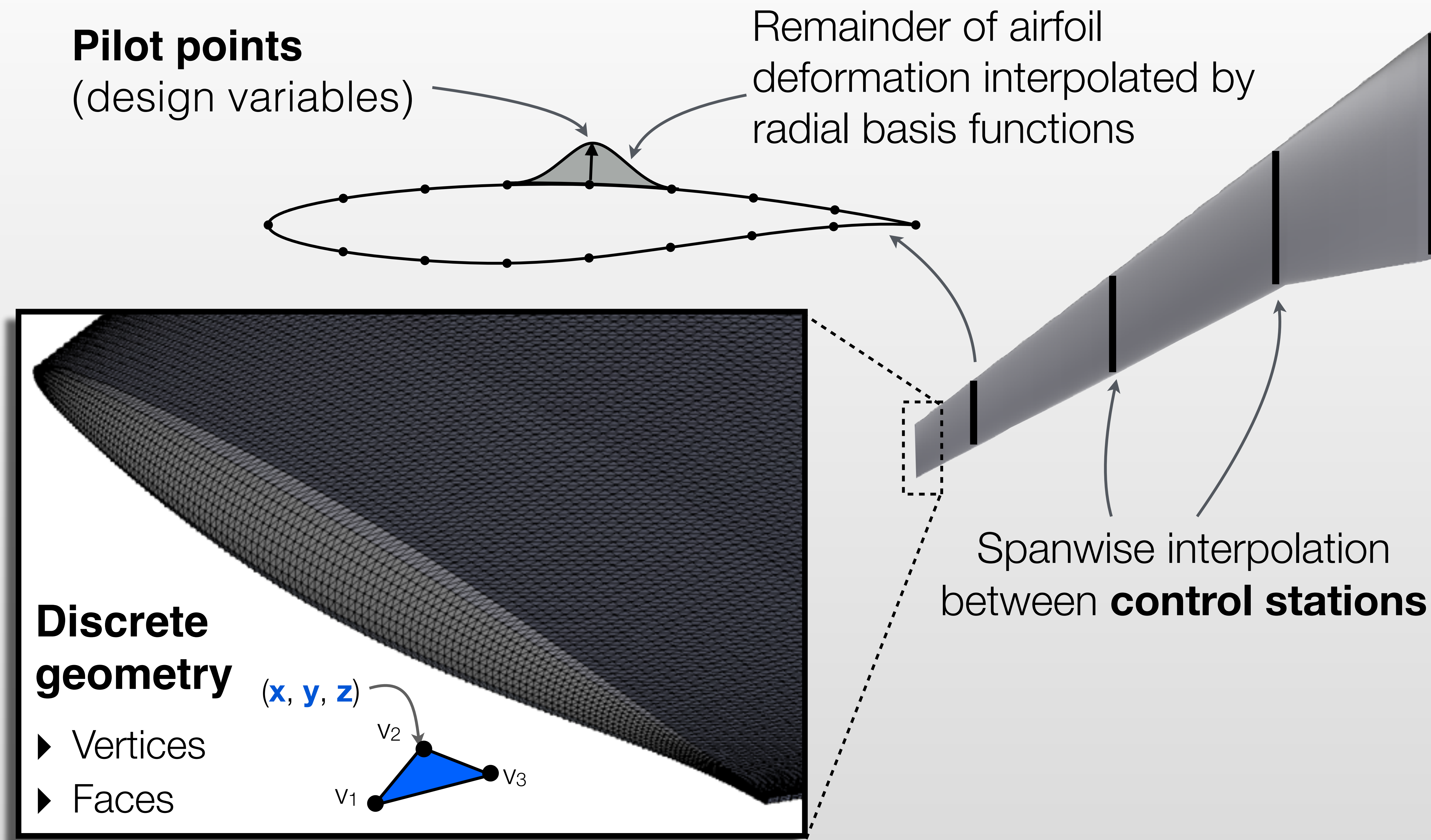
Objectives

- Demonstrate **adaptive shape control** system:
  - **Automate** shape control refinement.
  - **Accelerate** design improvement.
  - **Discover** the parameters necessary to improve a design.
  - Obtain better designs with less sensitivity to designer's decisions about parameterization.

Outline

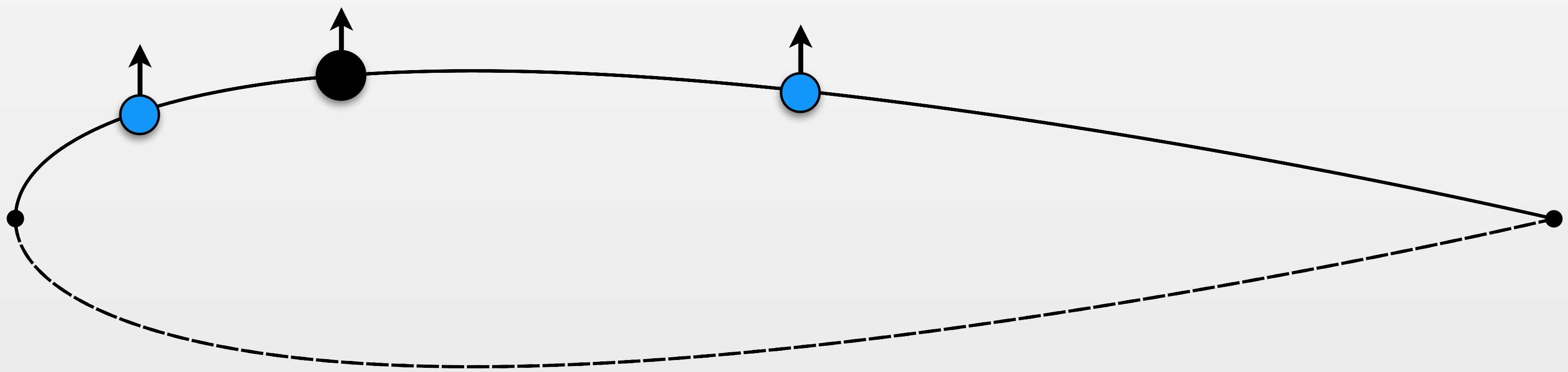
- ✓ Introduction
- **How** does refinement work?
  - Geometry modelers
  - Refinement mechanics
- **When** should refinement happen?
  - Pacing
  - *Example 1 — Transonic airfoil design*
- **Where** should the shape control be refined?
  - Adaptively choosing the best parameters
  - *Example 2 — Discovering necessary parameters*

Direct Manipulation



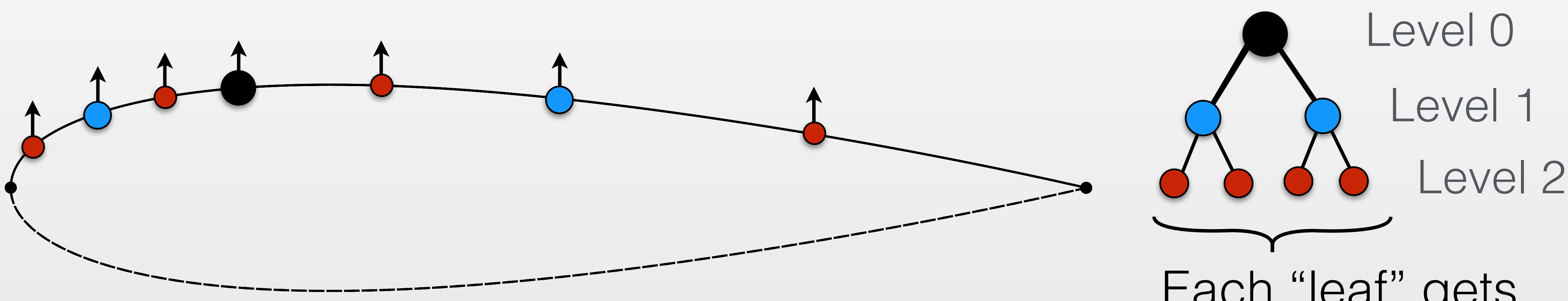
Refinement Mechanics

View shape parameterization as **binary tree:**



Refinement Mechanics

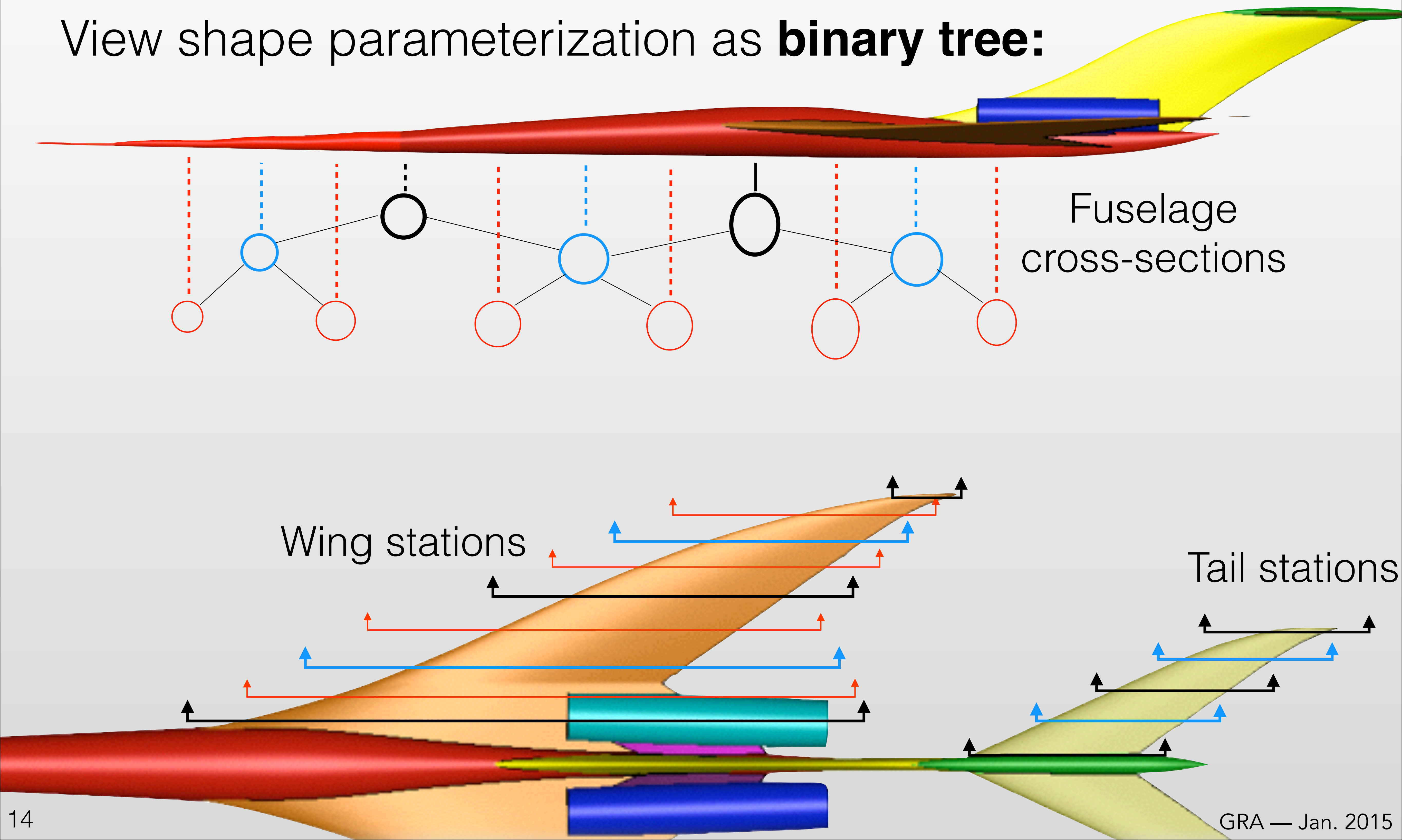
View shape parameterization as **binary tree:**



Each "leaf" gets two children

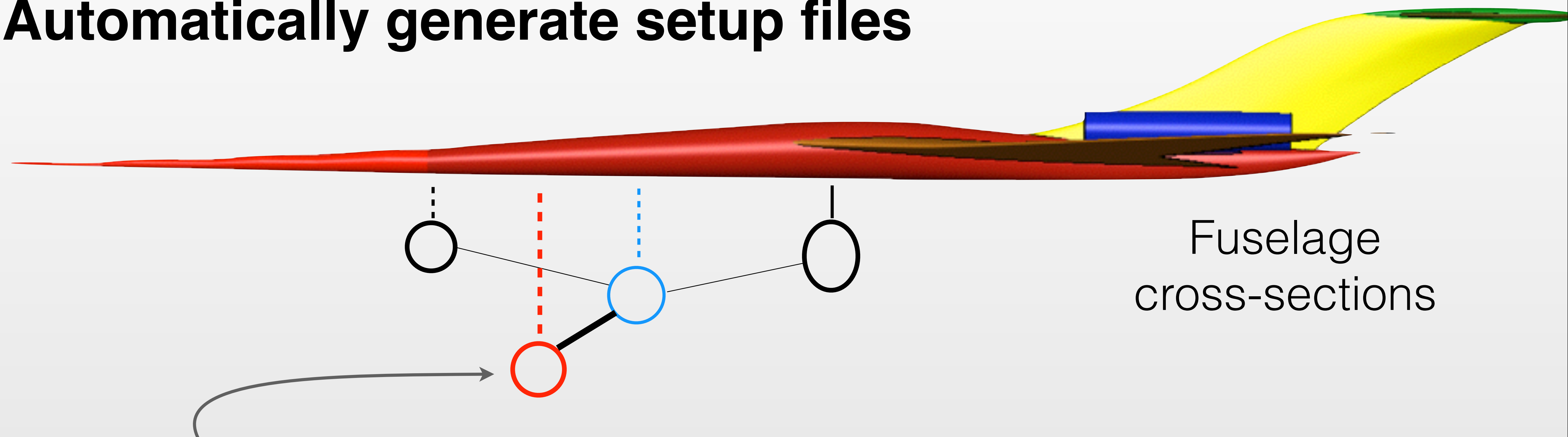
Refinement Mechanics

View shape parameterization as **binary tree:**



Automatic Refinement

Automatically generate setup files



1. Insert new parameter
2. Interpolate value
3. Transfer optimization parameters:
   - Min and max bounds
   - Scale factor

Outline

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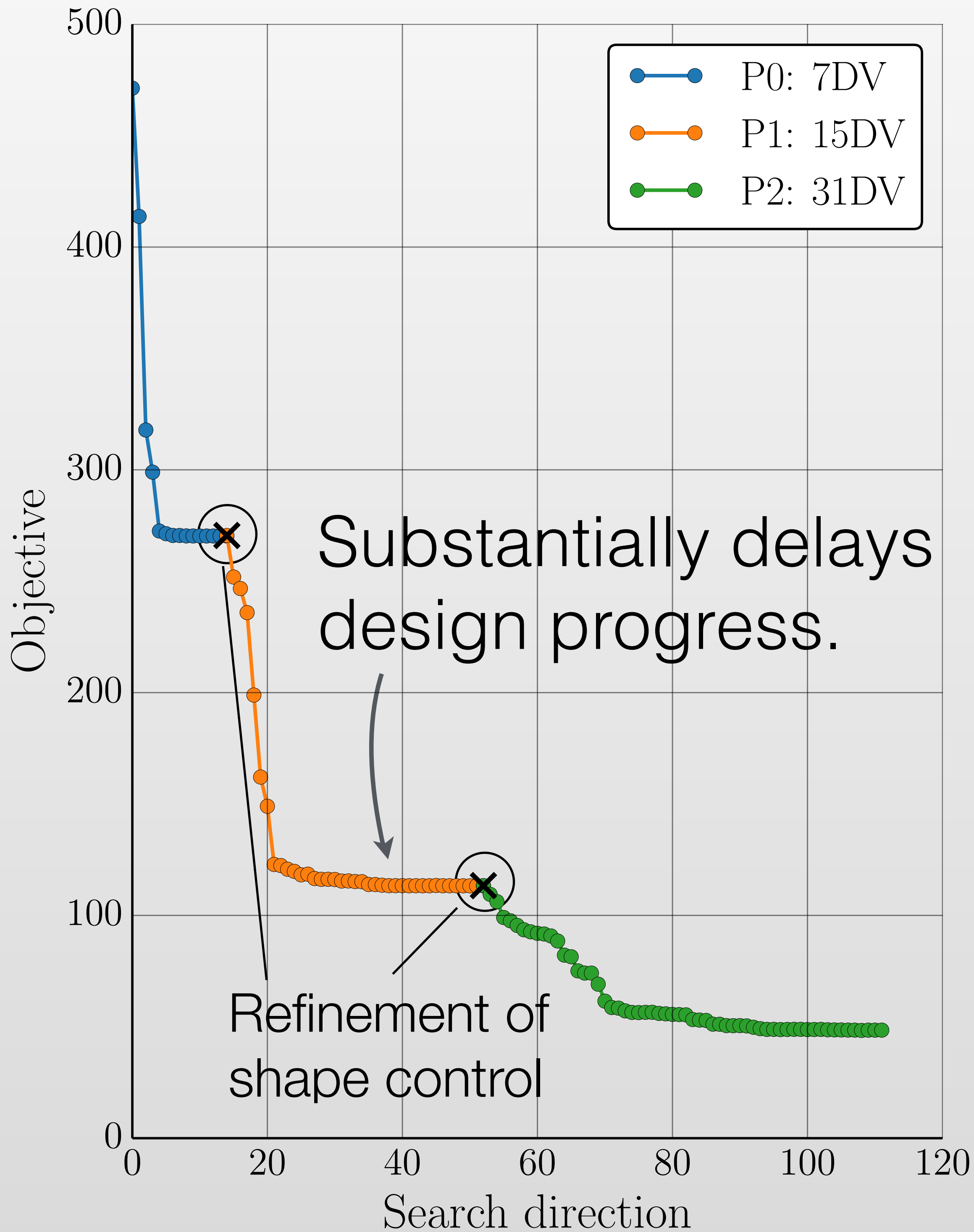
Pacing of Shape Control Refinement

Trigger: Automatic signal to transition to the next parameterization.



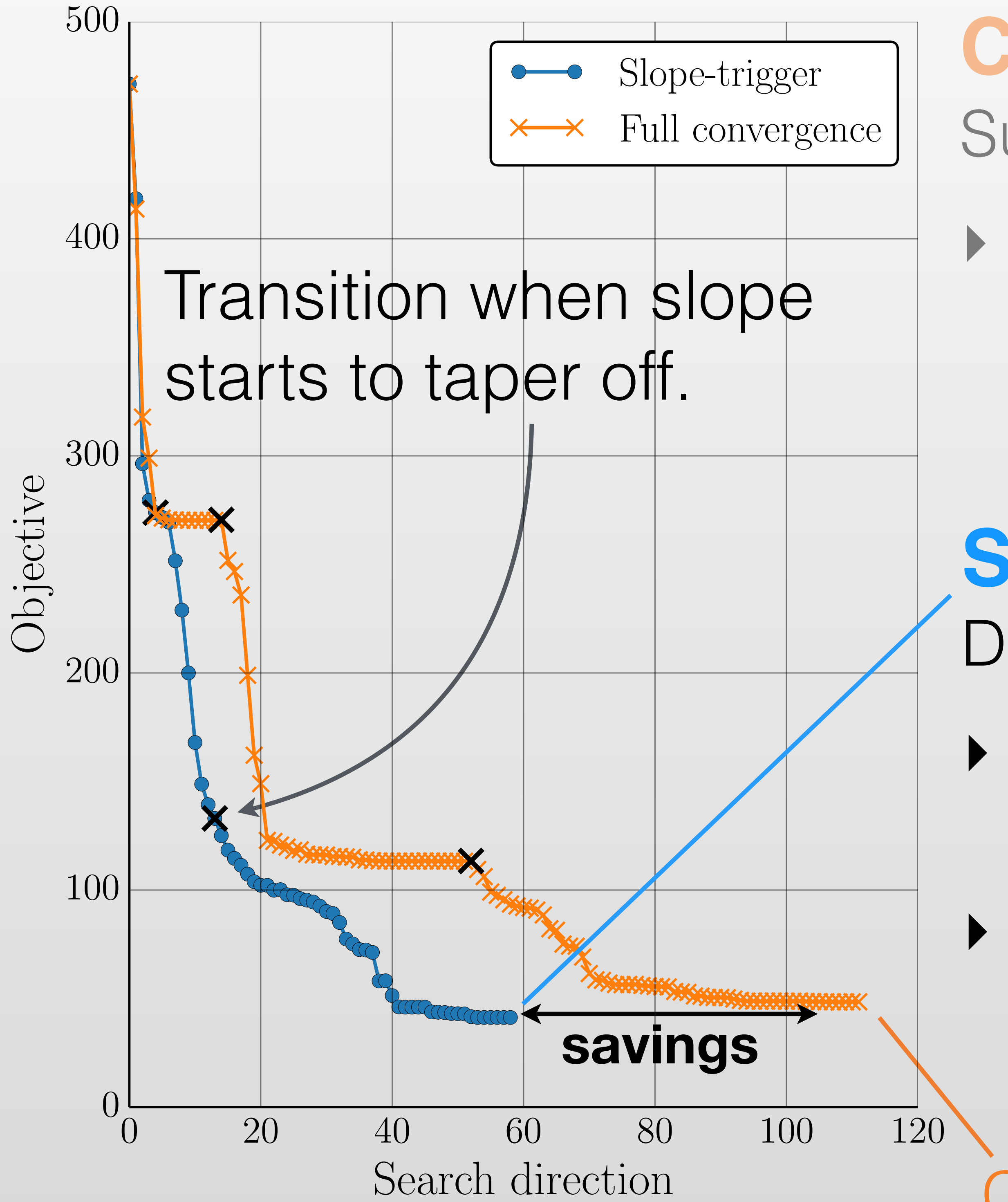
Convergence:

Sufficient reduction of gradients (or KKT)

- Indicates that nearly all expected design improvement *for this search space* has been attained.

Pacing of Shape Control Refinement

Trigger: Automatic signal to transition to the next parameterization.



Convergence:

Sufficient reduction

- Indicates that nearly all expected design improvement *space*

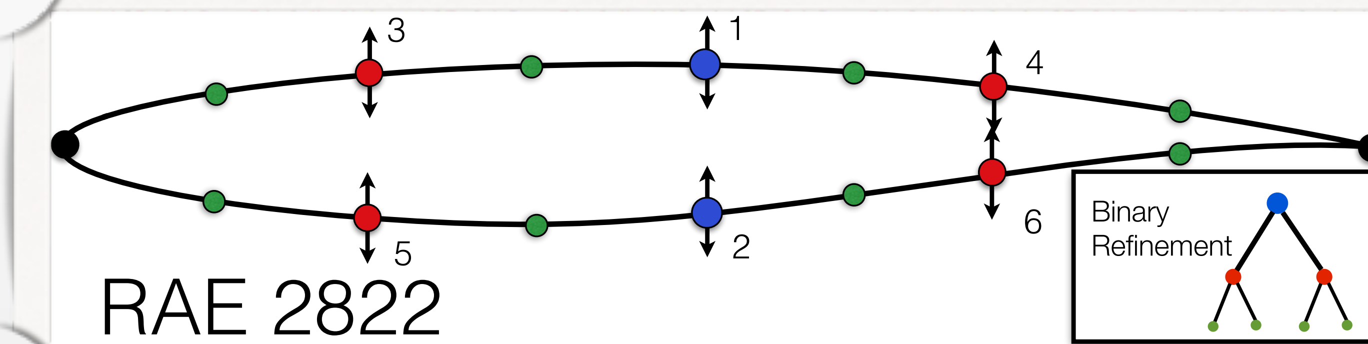
Slope Trigger:

Deceleration of design improvement

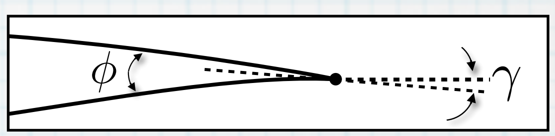
- Indicates diminishing returns on **computational** time.
- Avoids over-optimizing in coarse search spaces.

Example 1: Transonic Airfoil Design

Purpose: Demonstrate computational **acceleration** with automatic parameterization refinement.



$$\text{minimize } \mathcal{J} = C_{D_1} + C_{D_2}$$
$$\text{s.t. } C_{L_1} = C_{L_2} = 0.75$$
$$C_{M_1} \geq -0.18 \ (\mathbf{V})$$
$$C_{M_2} \geq -0.25 \ (\mathbf{V})$$
$$9^\circ \leq \phi \leq 13^\circ$$



$$\gamma \leq 6^\circ \ (\mathbf{V})$$
$$A \geq A_{RAE} \approx 0.07787$$
$$t_i \geq 0.9 t_{RAE_i} \forall i$$

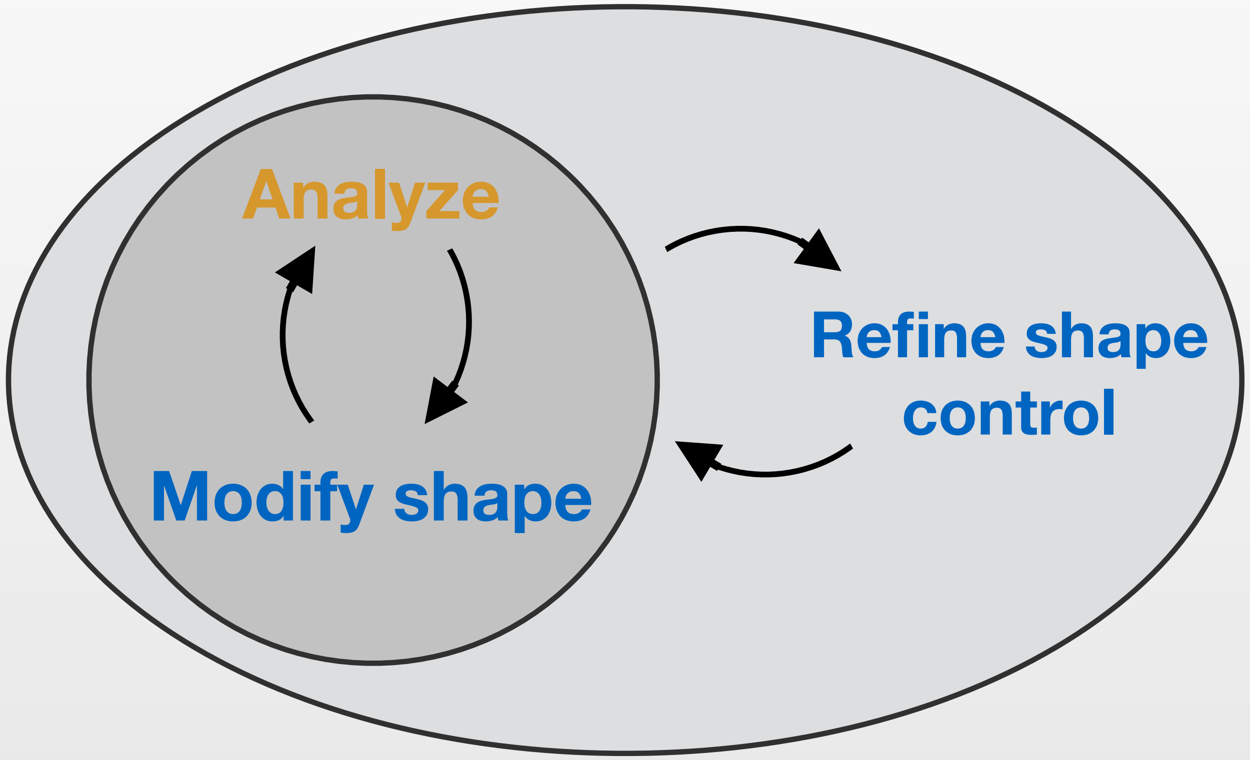
Objective: Minimize drag at Mach 0.79 and Mach 0.82

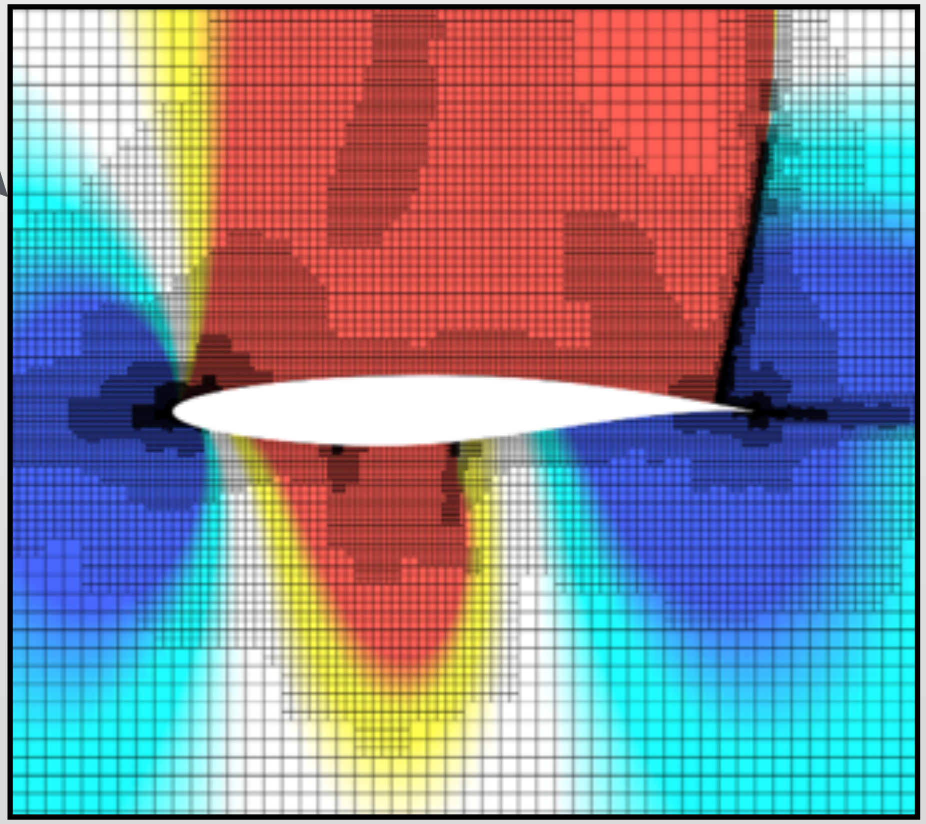
27 Constraints:

- (2) Fixed Lift
- (2) Min. Pitching moment
- Min. Area
- (20) Min. 90% thickness
- Trailing edge camber line
- Boat-tail angle

Cart3D Adjoint-based Design Framework

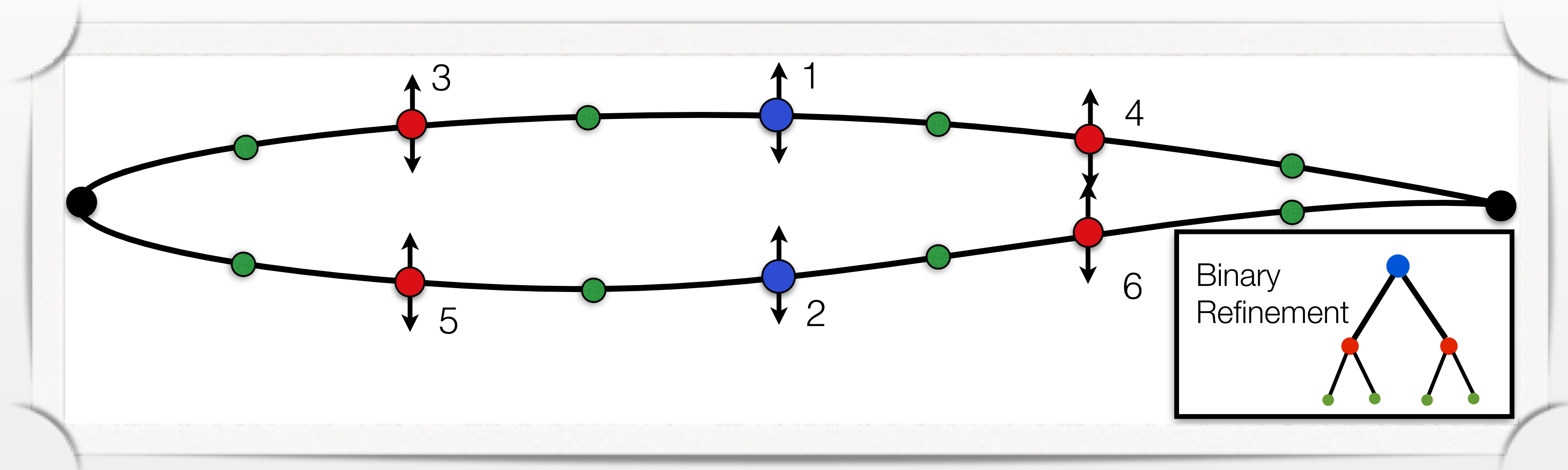
- Inviscid Cartesian cut-cell method
- Aerodynamic objective and constraint gradients via adjoints
- Geometric constraints differentiated analytically
- SNOPT SQP optimizer
- Adjoint-driven mesh adaptation



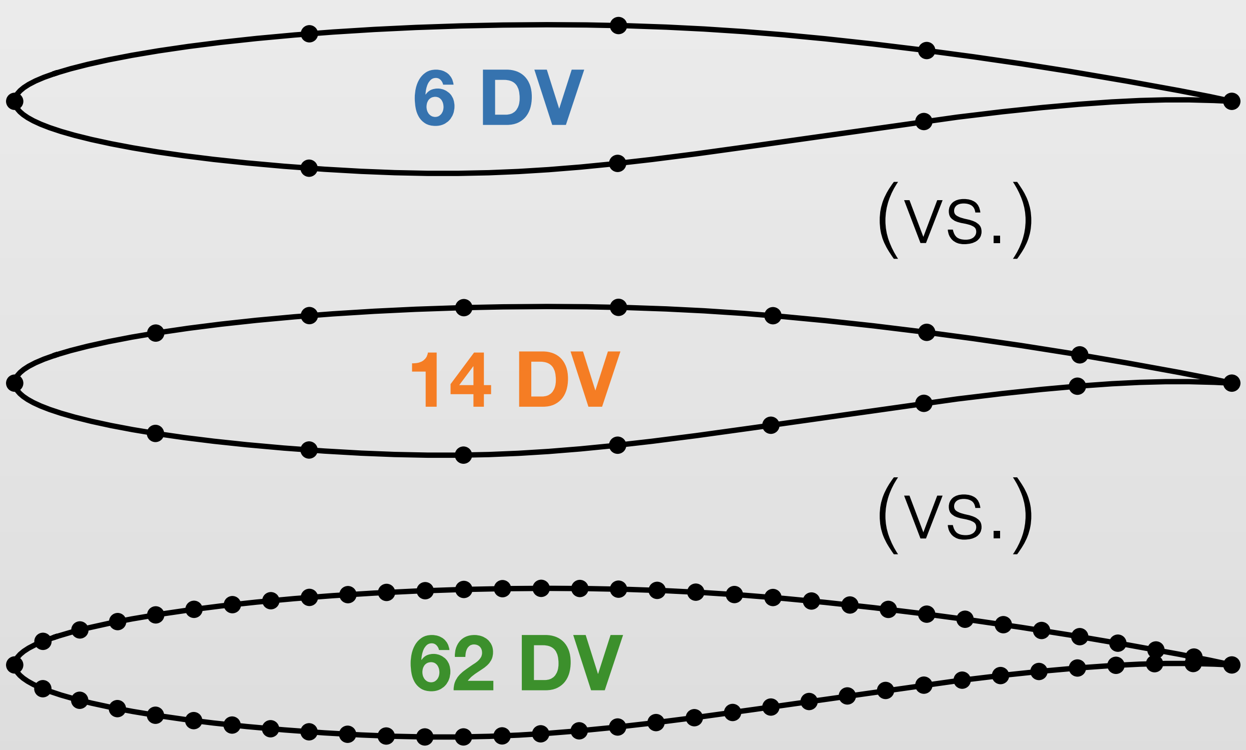


Parameterization

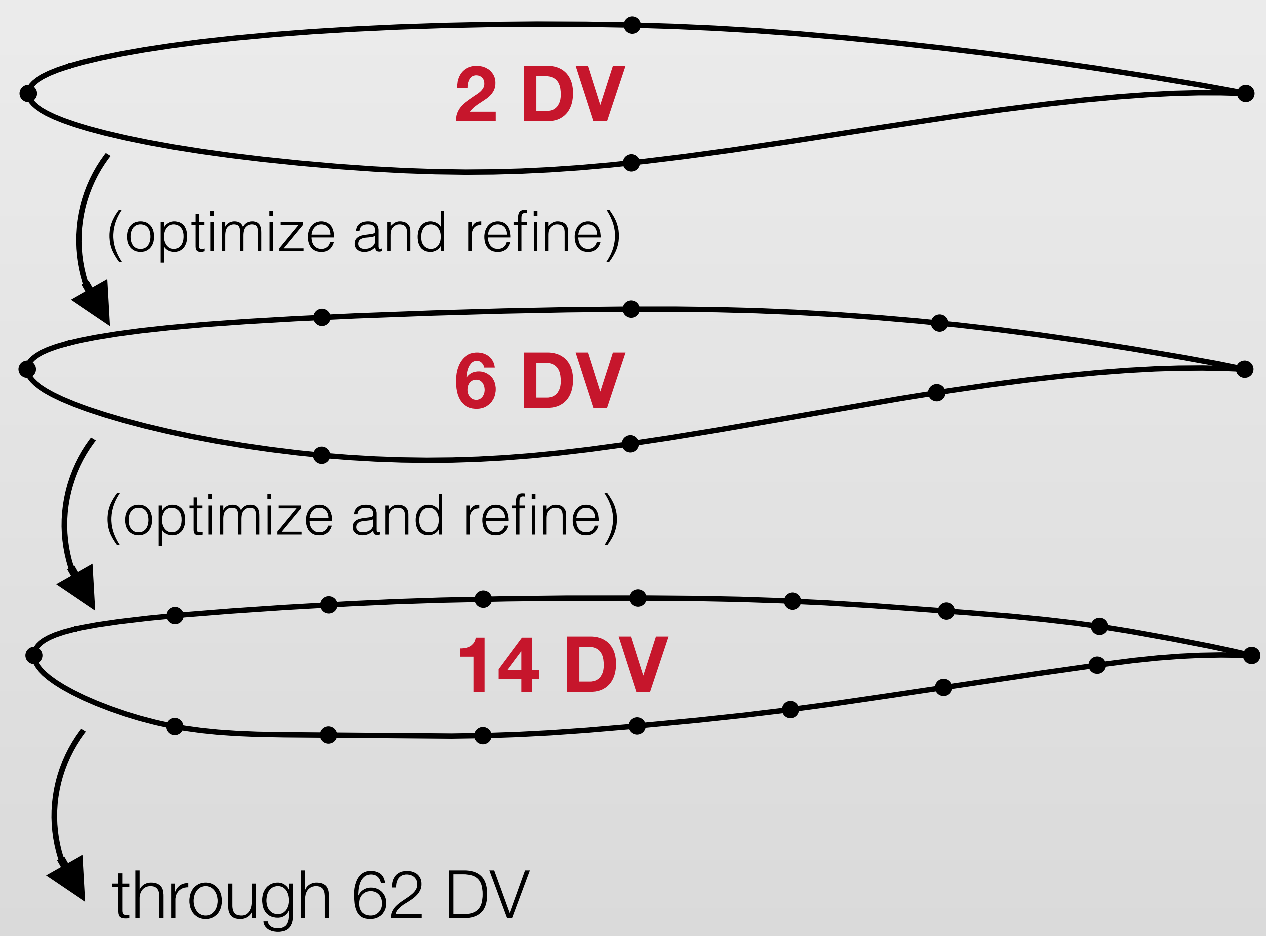
Curve parameterization with direct manipulation



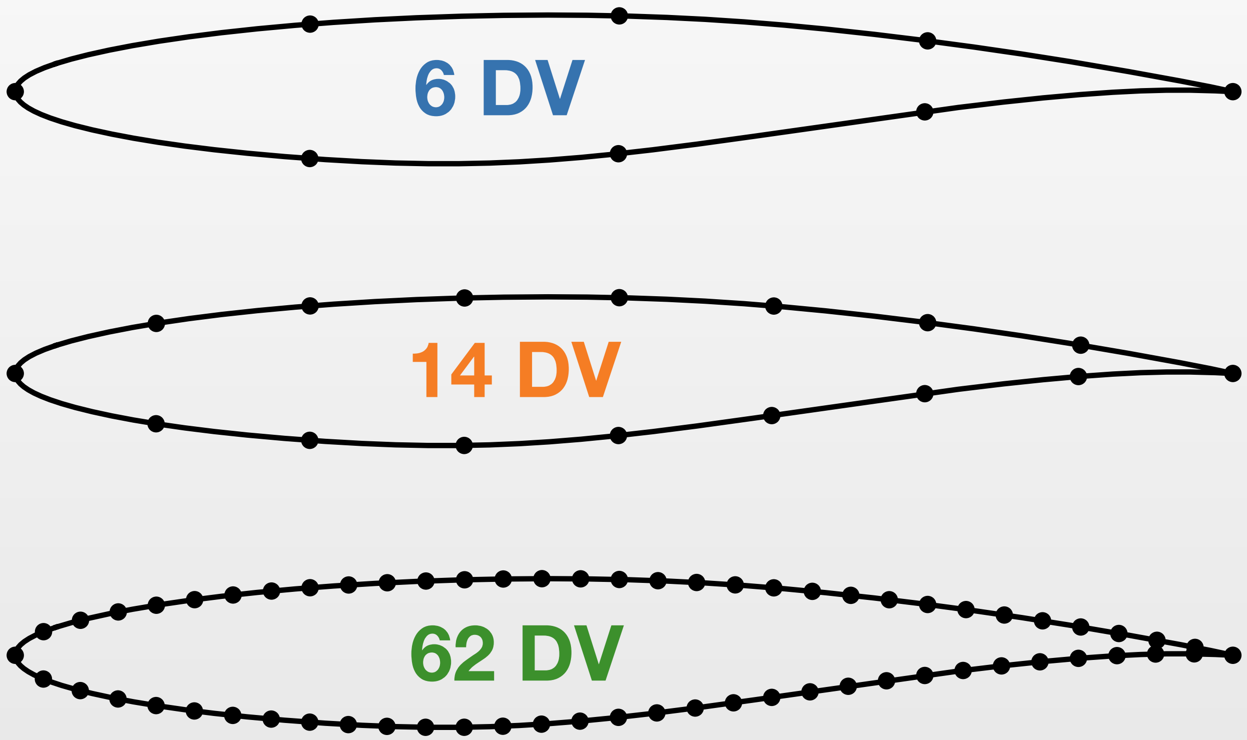
Consider 3 **static** parameterizations



Compare to uniform **progressive** refinement



Static Parameterizations

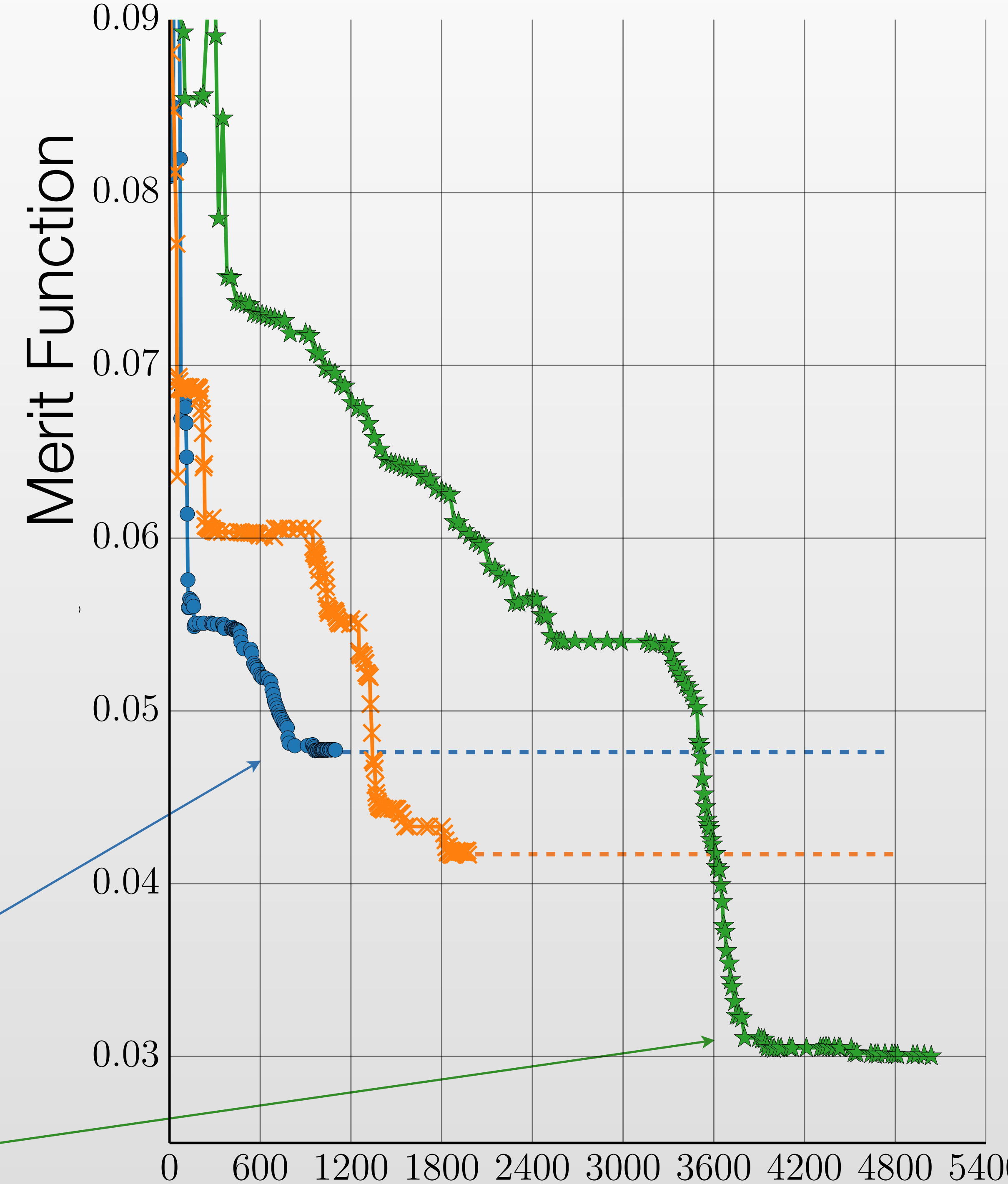


Low resolution:

Faster design improvement

High resolution:

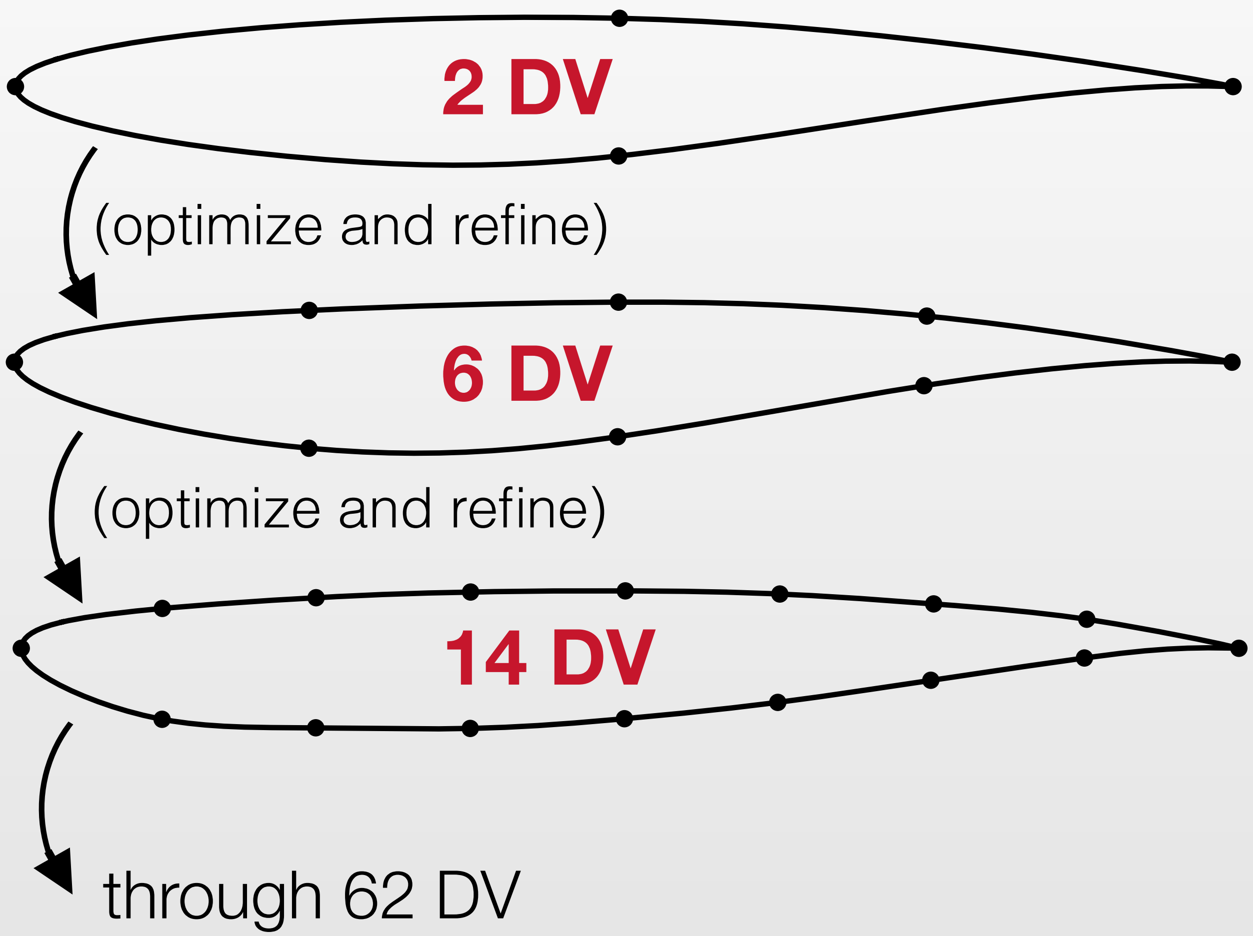
Better designs



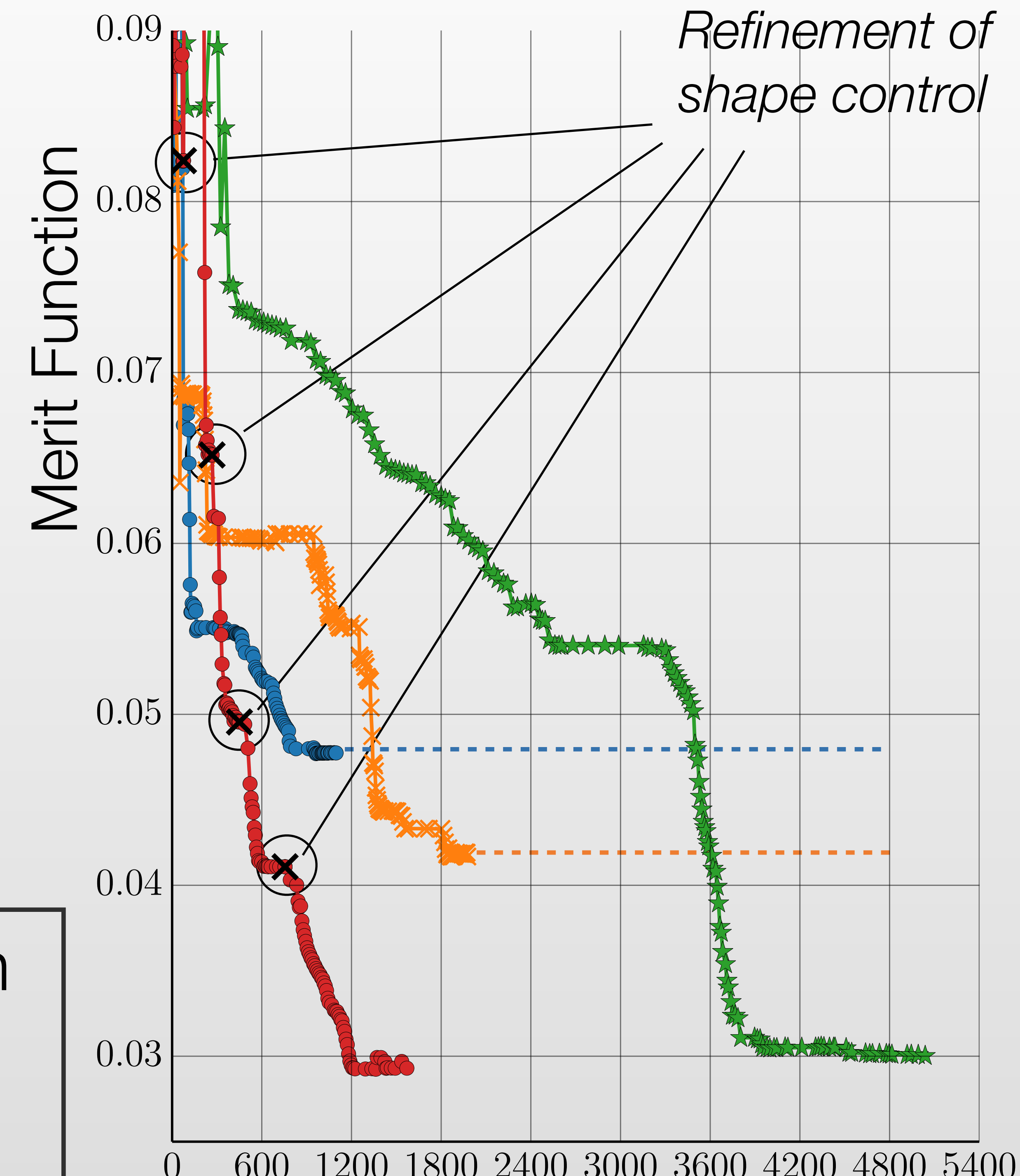
Wall clock time

In minutes, plotted at major search iterations, on 20 Ivybridge cores

Progressive Parameterization



Fast improvement in coarse search spaces, but ultimately approaching **full design space**.



Wall clock time

In minutes, plotted at major search iterations, on 20 Ivybridge cores

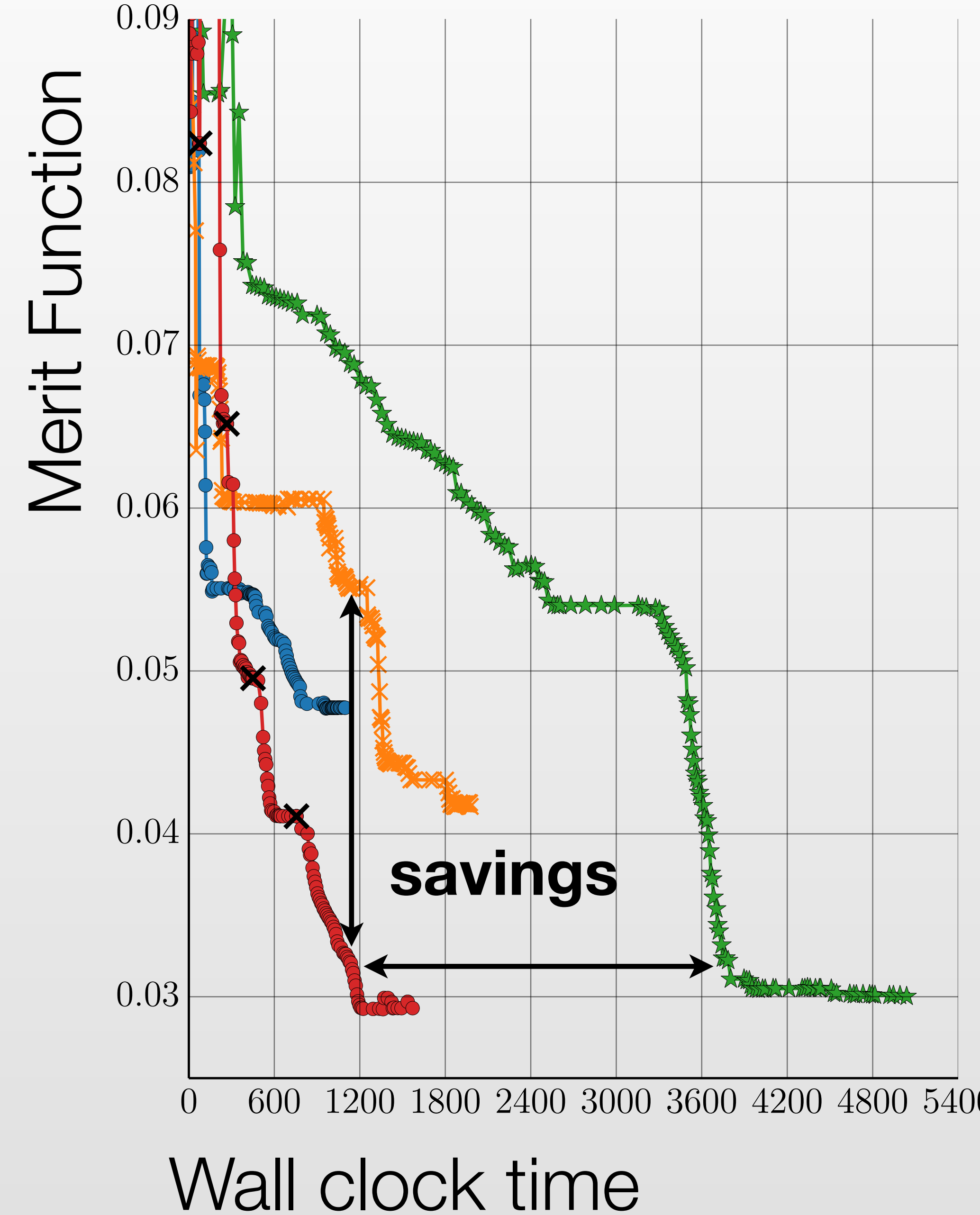
Cost

Factors contributing to acceleration:

- Early on there are few design variables:
  - Accelerates **BFGS rate of improvement** w.r.t search direction.
  - Reduces # of shape sensitivities and gradient projections.
- Later, more design variables are added, **preventing optimization from stalling.**

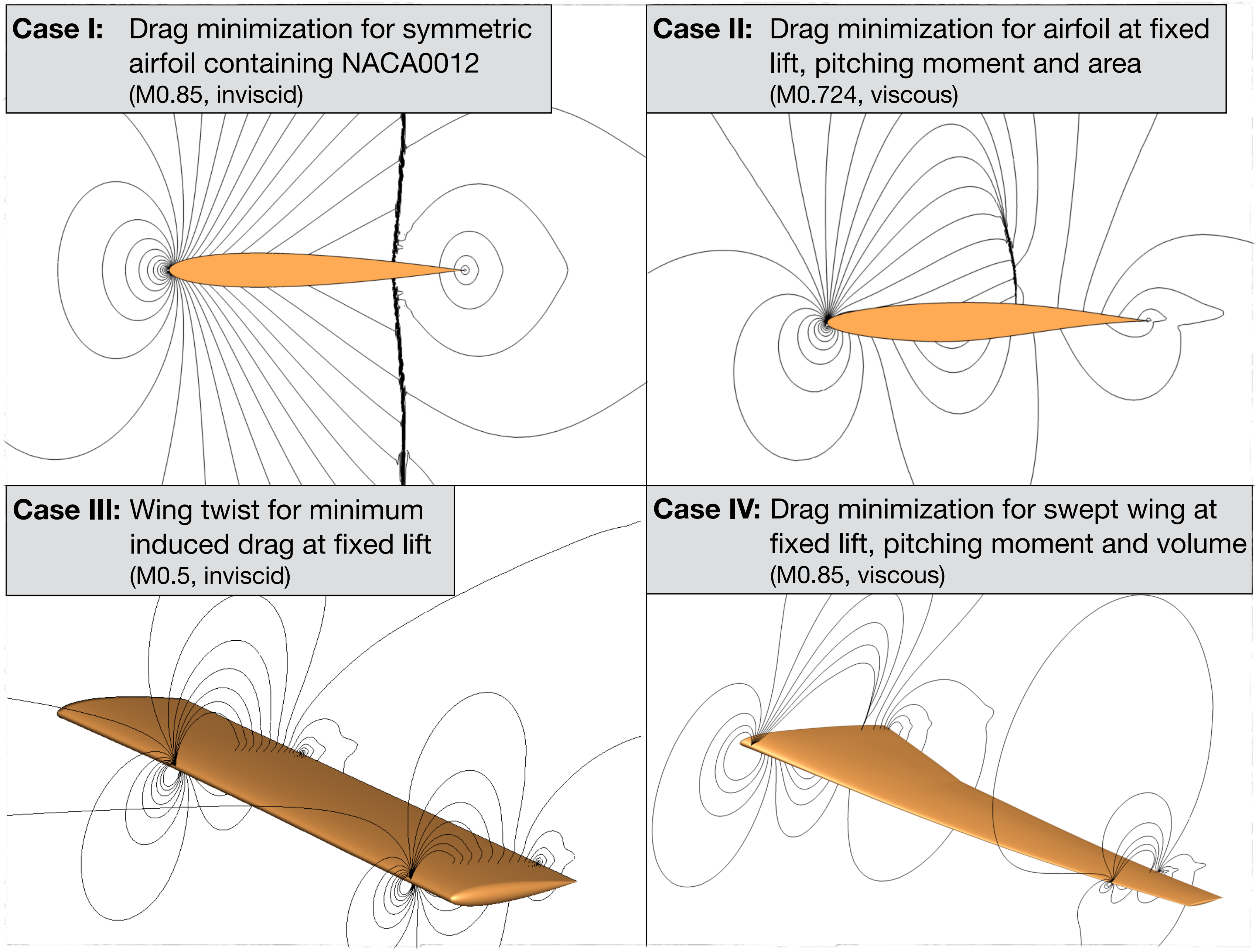
Cost per design iteration: 4-8 minutes

- Geometry generation
- N_{DV} shape derivative computations
- 2 adaptively meshed flow solutions
- 6 adjoints (drag, lift, pitching moment)
- 29 N_{DV} gradient projections



In minutes, plotted at major search iterations, on 20 Ivybridge cores

Optimization Benchmarks



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Four optimization benchmarks using progressive parameterization

Outline

- ✓ Introduction
- ✓ How does refinement work?
  - Geometry modelers
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- ✓ When should refinement happen?
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  - *Example 1 — Transonic airfoil design*
- **Where** should the shape control be refined?
  - Adaptively choosing the best parameters
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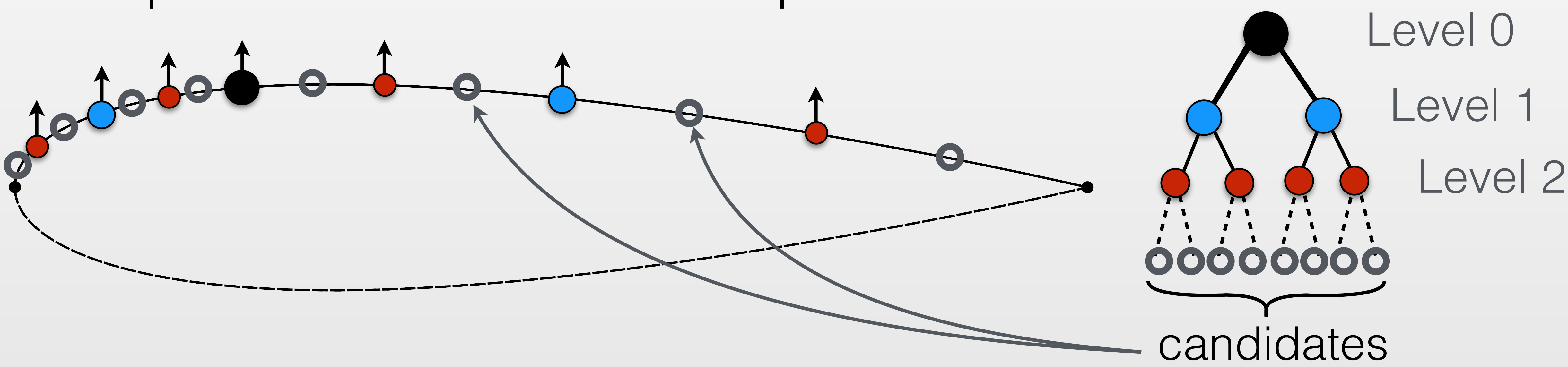
Candidate Shape Parameters

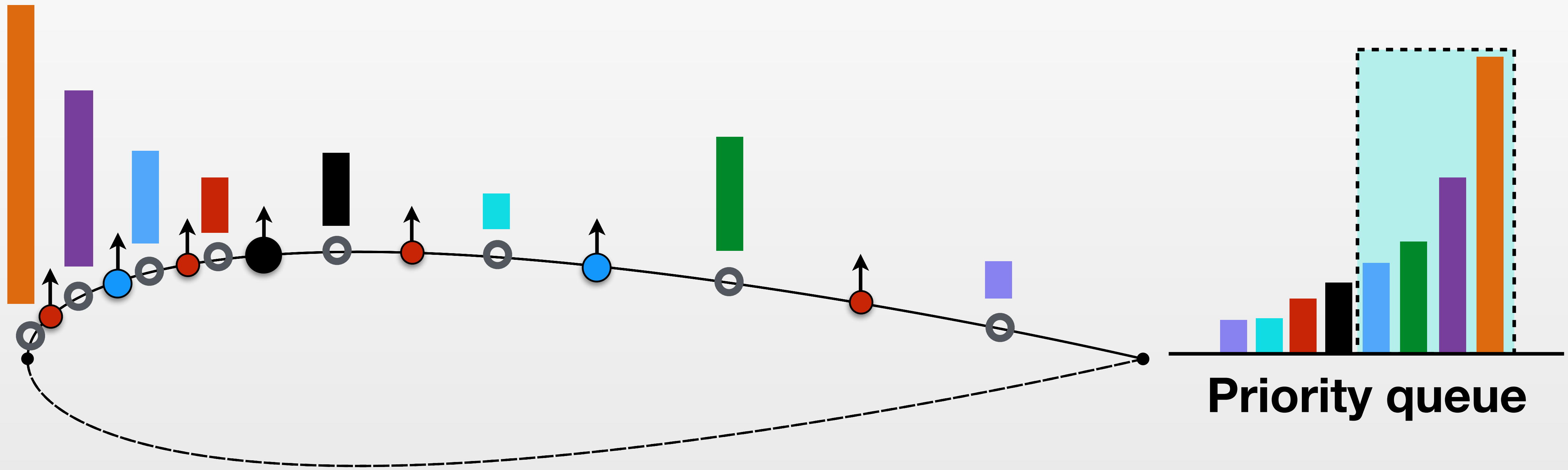
Add the **most important** shape control.

- **Goal:** Further accelerate design by using a more optimal **distribution** of shape control.



1. Modeler provides a list of possible shape control refinement locations.

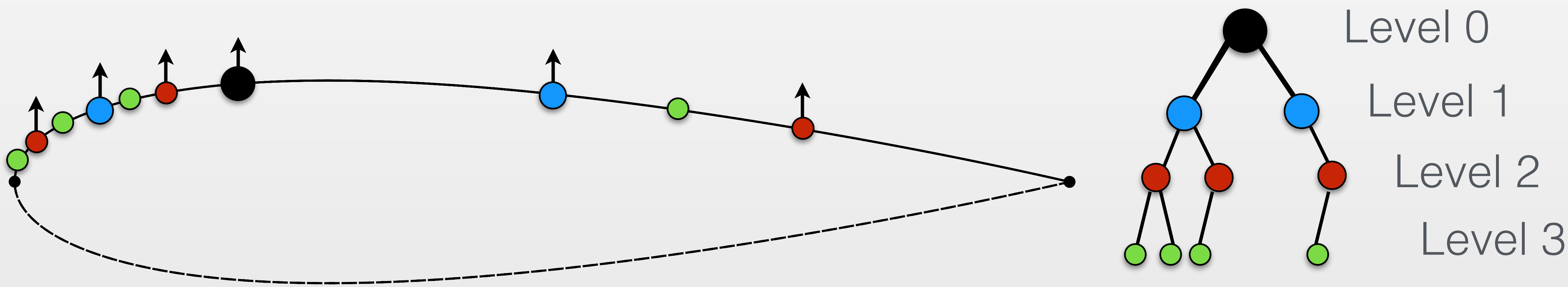
Adaptive Refinement



1. Modeler provides a list of possible shape control refinement locations.

2. Rank candidates by relative "importance".

Candidate Shape Parameters



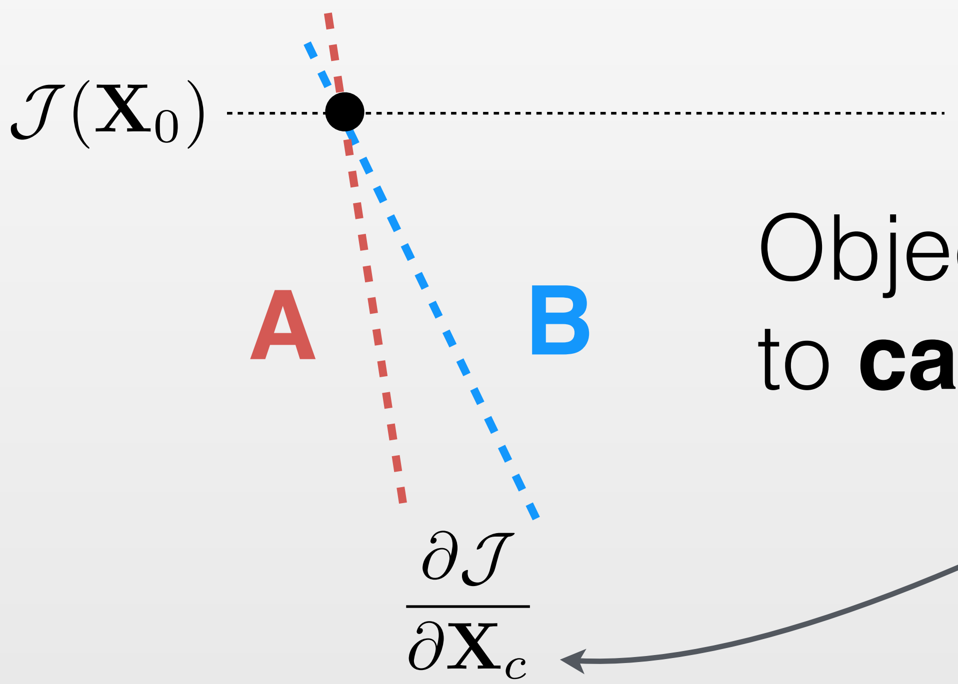
1. Modeler provides a list of possible shape control refinement locations.

2. Rank candidates by relative “importance”.

3. Selectively refine most important regions.

Effectiveness Indicator

Rank parameters by **ability to improve design.**

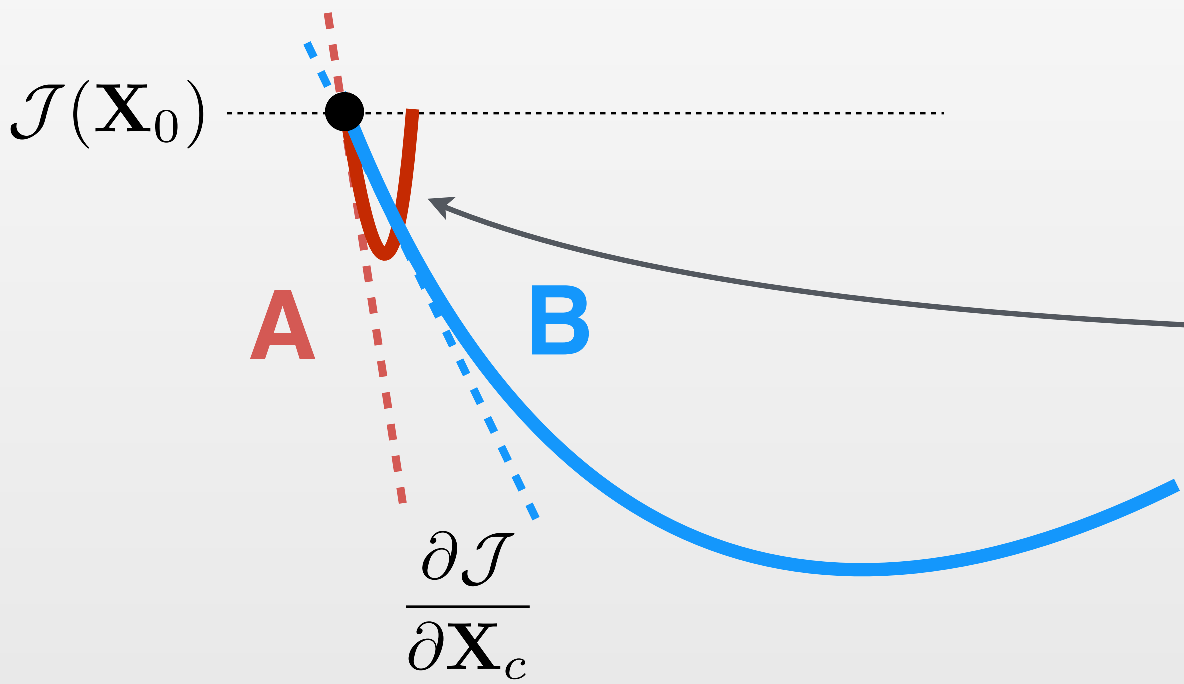


Objective gradients with respect to **candidate** design variables

Low extra cost:
Compute by projecting new shape derivatives into **existing** adjoint solutions.

Effectiveness Indicator

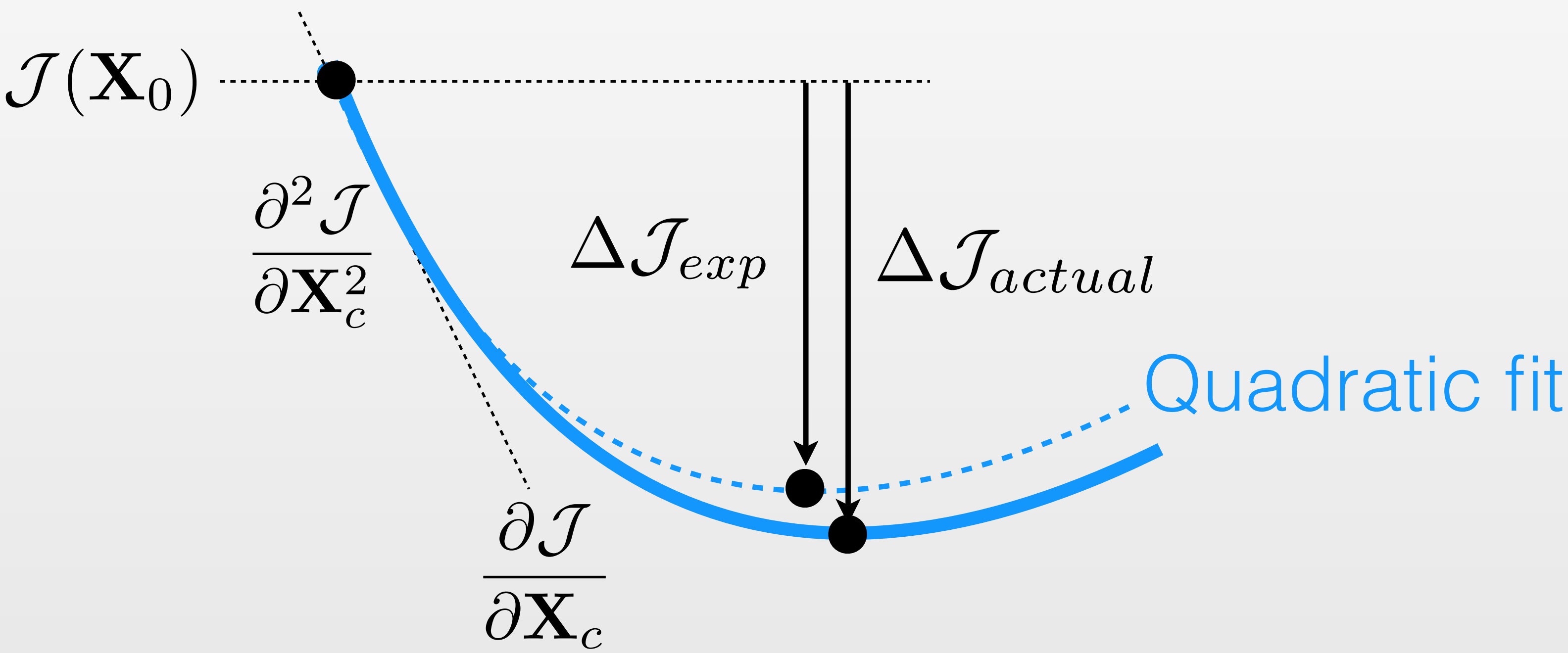
Rank parameters by **ability to improve design.**



Gradients alone can be misleading, especially for poorly scaled problems.

Effectiveness Indicator

Rank parameters by **ability to improve design.**



Minimizer gives **expected design improvement:**

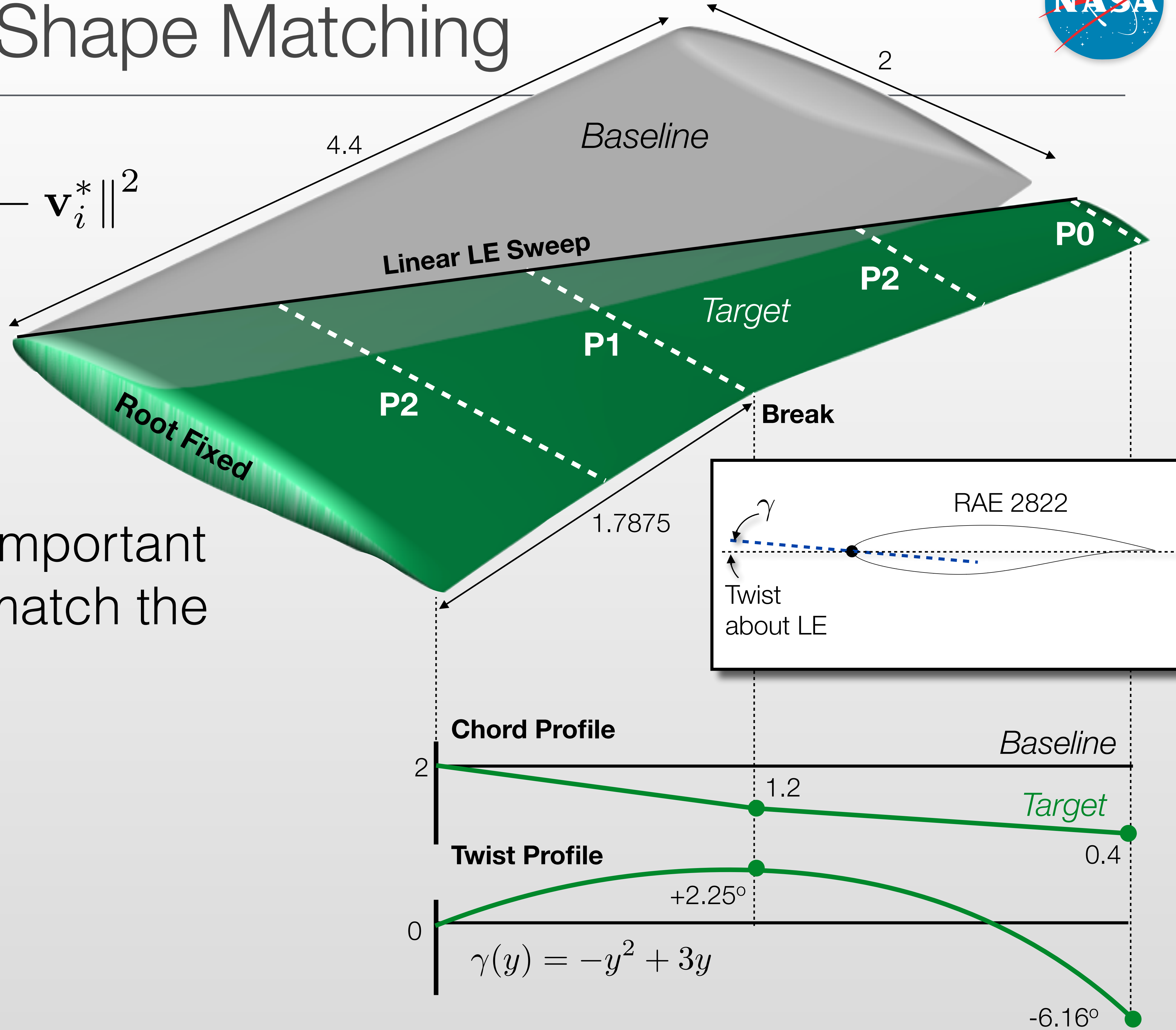
$$\Delta \mathcal{J}_{exp}(\mathbf{C}_c) = \frac{1}{2} \frac{\partial \mathcal{J}}{\partial \mathbf{X}_c}^T \left(\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} \right)^{-1} \frac{\partial \mathcal{J}}{\partial \mathbf{X}_c}$$

Gradients

Hessian

Geometric Shape Matching

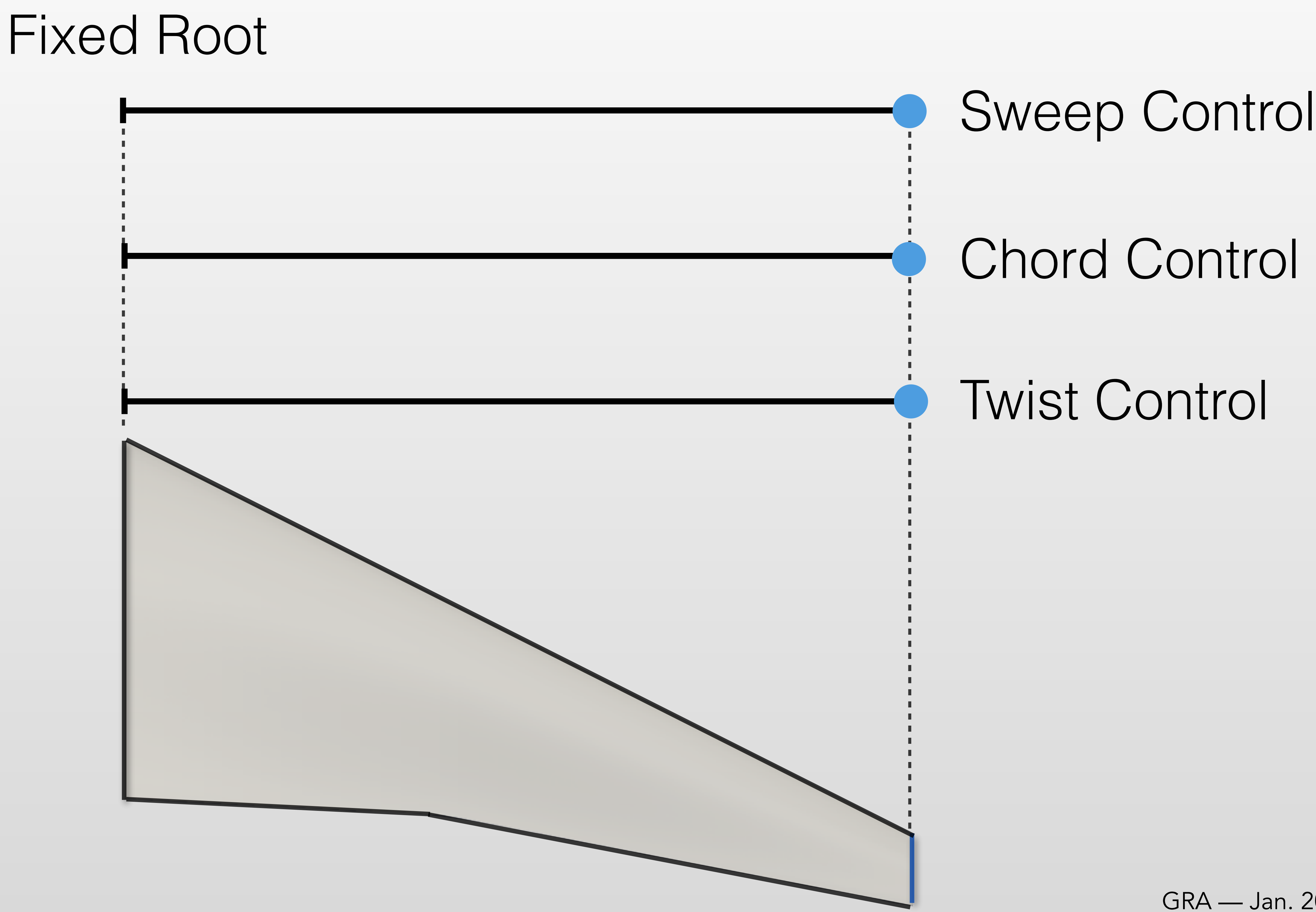
$$\mathcal{J} = \sum_{i=1}^{N_{verts}} \left\| \mathbf{v}_i - \mathbf{v}_i^* \right\|^2$$



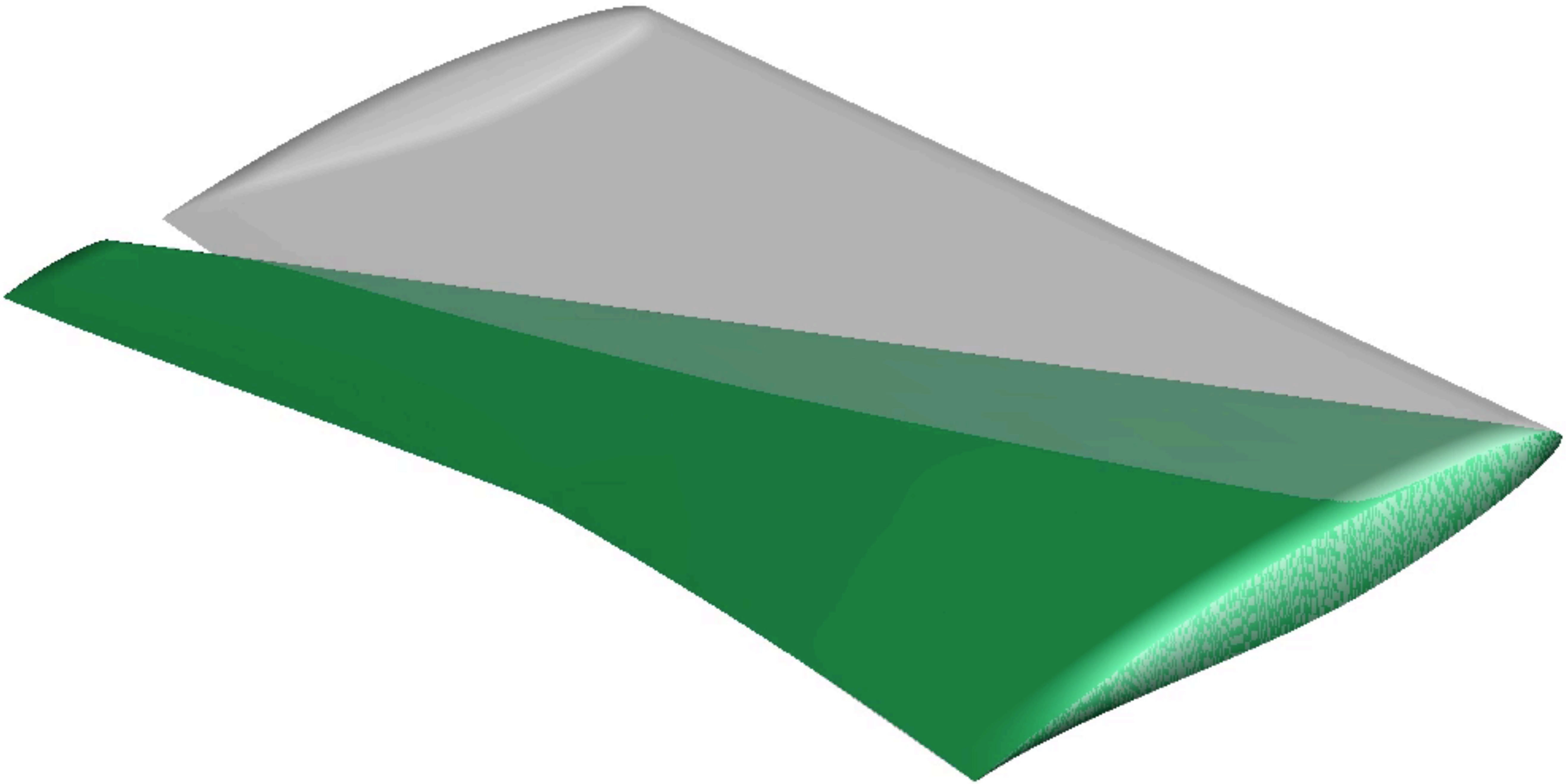
Goal:

Discover most important parameters to match the target shape

Initial Shape Parameters



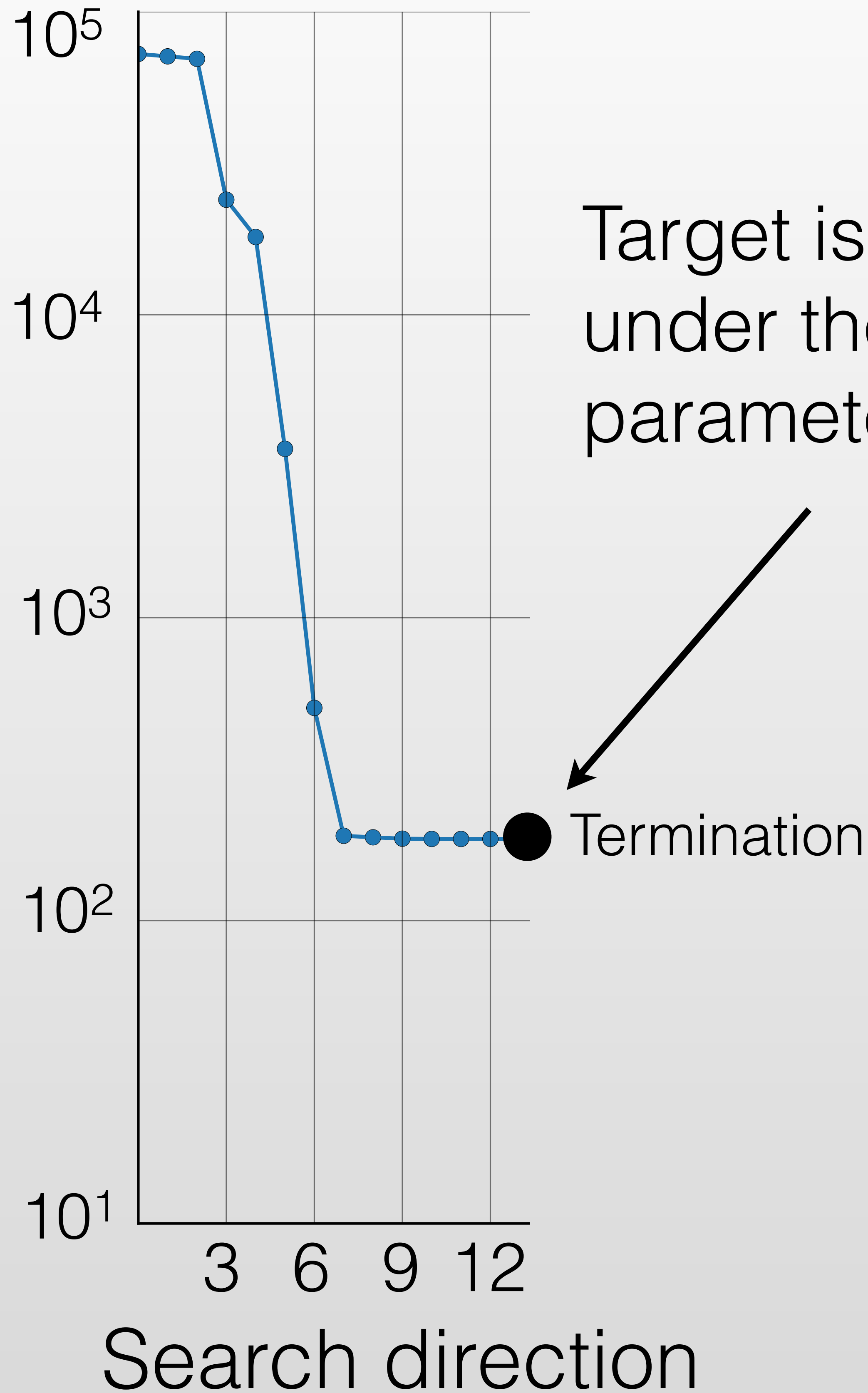
Shape Matching under Initial Parameterization



Initial Optimization

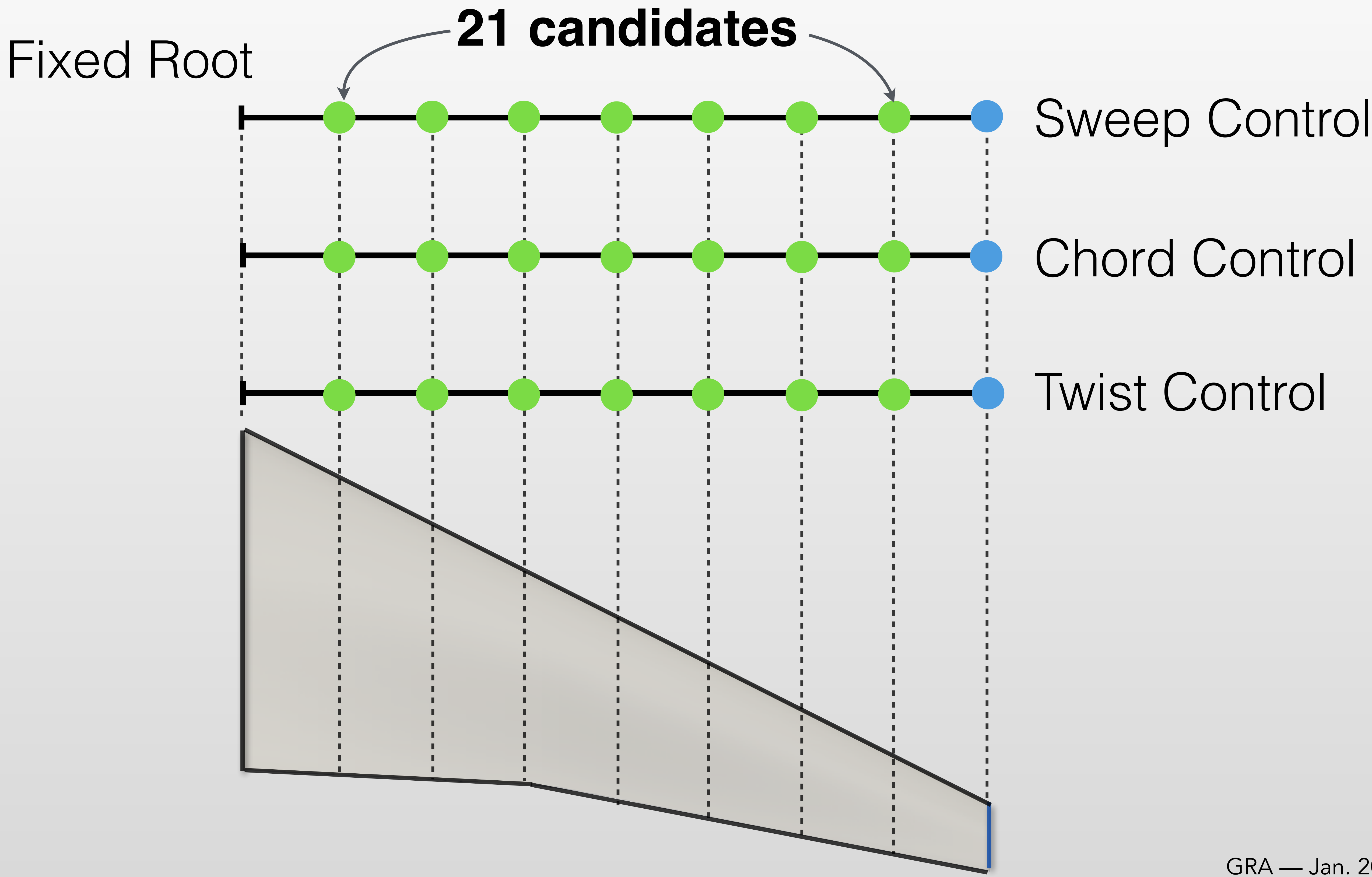
Objective

$$\mathcal{J} = \sum_{i=1}^{N_{verts}} \|\mathbf{v}_i - \mathbf{v}_i^*\|^2$$



Target is **unattainable** under the initial 3-DV parameterization.

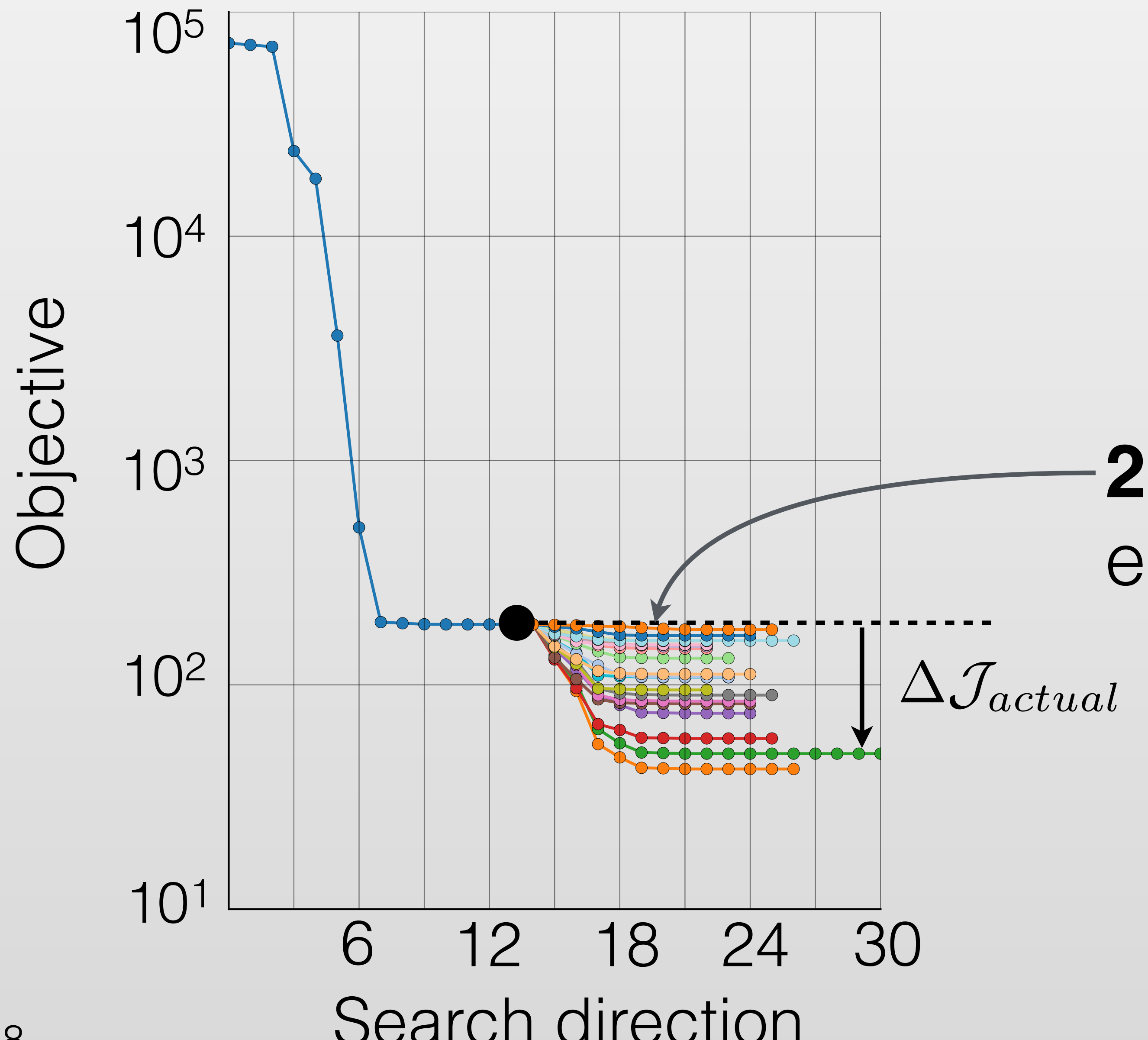
Adapt Shape Parameters

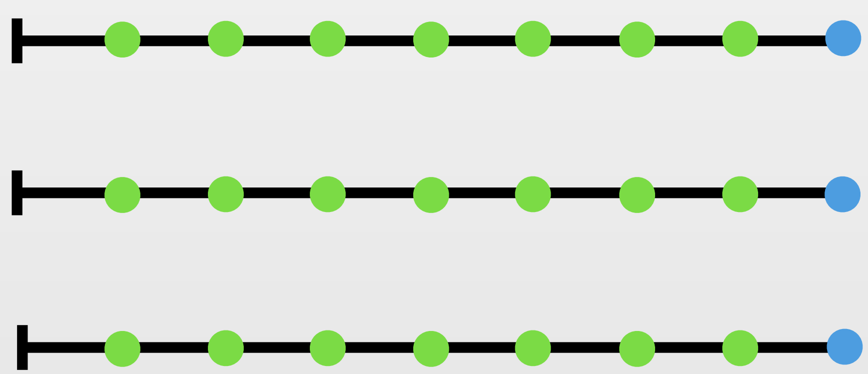


Indicator Validation

For each candidate:

1. **Predict** design improvement (via indicators)
2. Measure **actual** design improvement (run optimization)





21 lines correspond to adding each one of the candidates.

Compute Indicator

$$\Delta \mathcal{J}_{exp}(\mathbf{C}_c) = \frac{1}{2} \frac{\partial \mathcal{J}}{\partial \mathbf{X}_c}^T \left(\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} \right)^{-1} \frac{\partial \mathcal{J}}{\partial \mathbf{X}_c}$$

Transformation from surface derivatives to **design variables**:

$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} = \frac{\partial \mathcal{J}}{\partial \mathbf{S}} \frac{\partial^2 \mathbf{S}}{\partial \mathbf{X}_c^2} + \frac{\partial \mathbf{S}}{\partial \mathbf{X}_c}^T \frac{\partial^2 \mathcal{J}}{\partial \mathbf{S}^2} \frac{\partial \mathbf{S}}{\partial \mathbf{X}_c}$$

0 for linear deformers

Shape derivatives

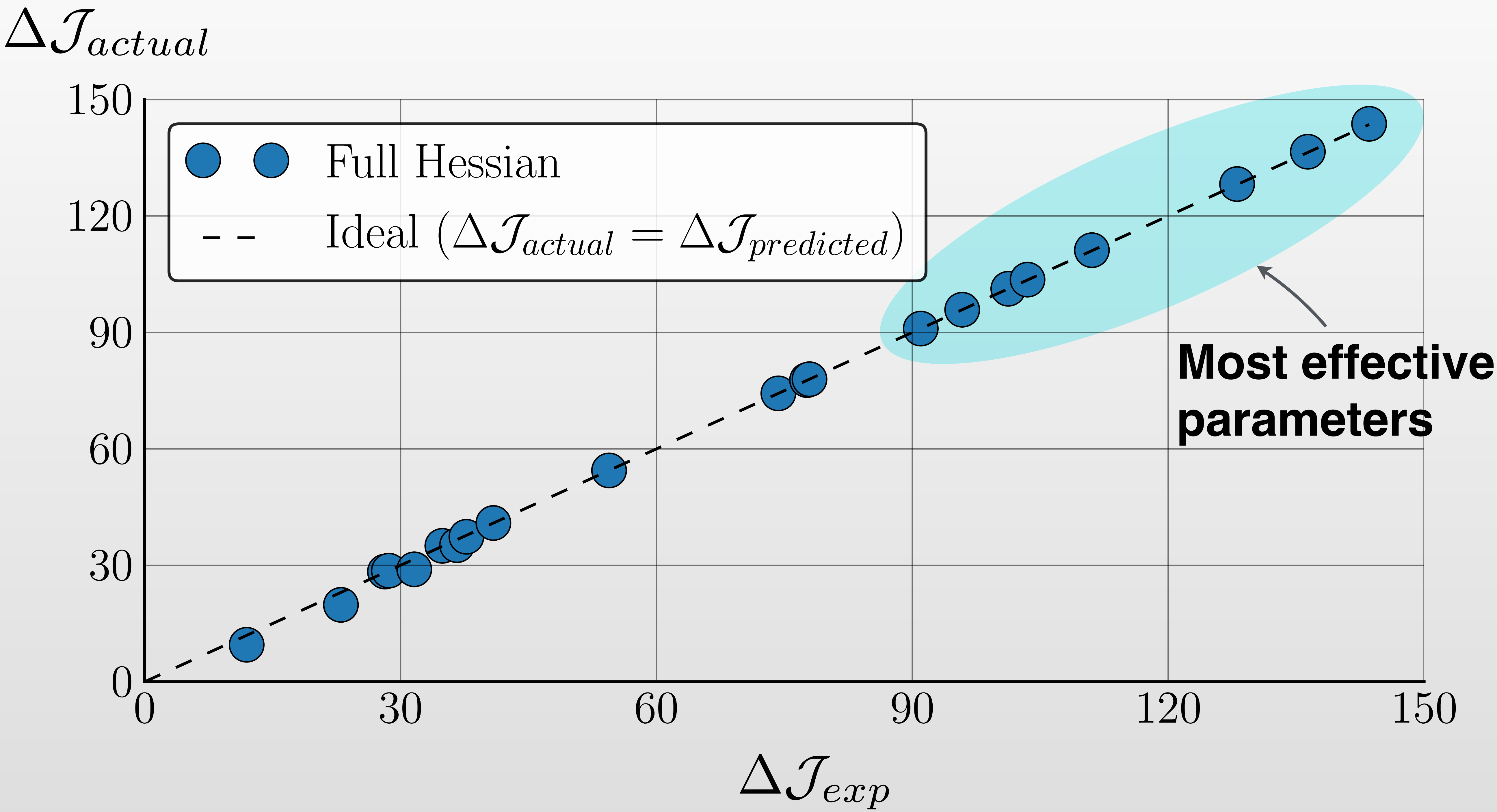
Shape matching

$$\mathcal{J} = \sum_{i=1}^{N_{verts}} \|\mathbf{v}_i - \mathbf{v}_i^*\|^2$$

Surface Hessian known analytically:

$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{S}^2} = 2\mathbf{I}$$

Full Hessian Correlation

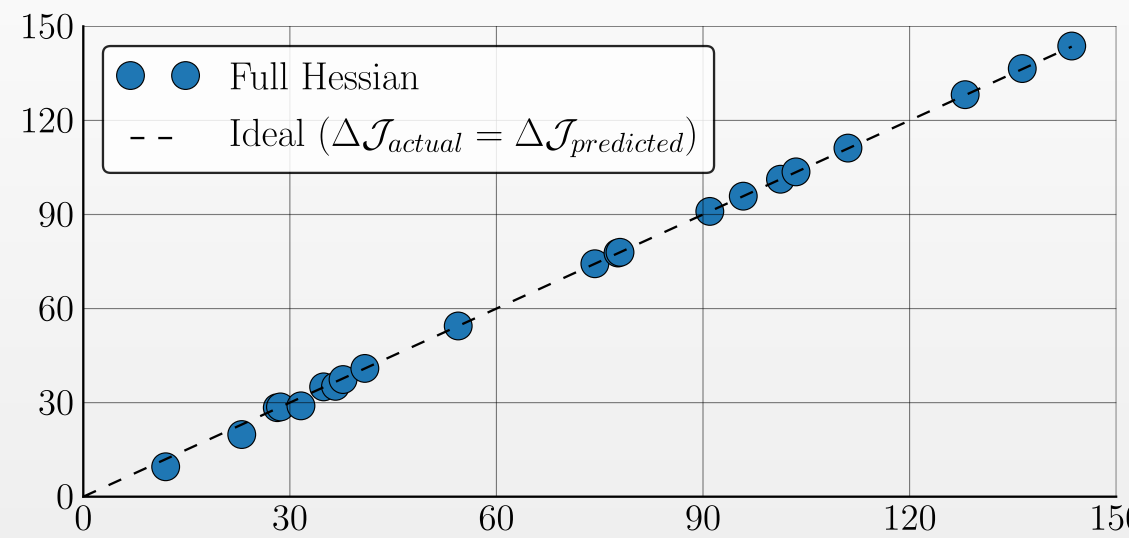


Approximations

Exact Hessian

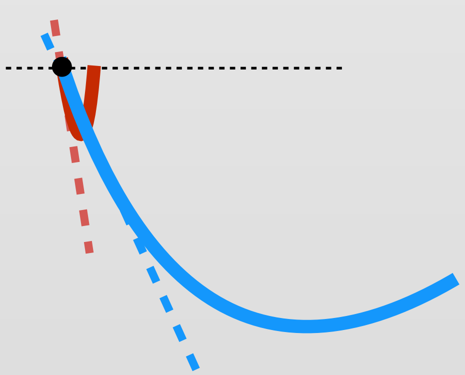
$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} = \frac{\partial \mathcal{J}}{\partial \mathbf{S}} \frac{\partial^2 \mathbf{S}}{\partial \mathbf{X}_c^2} + \frac{\partial \mathbf{S}^T}{\partial \mathbf{X}_c} \frac{\partial^2 \mathcal{J}}{\partial \mathbf{S}^2} \frac{\partial \mathbf{S}}{\partial \mathbf{X}_c}$$

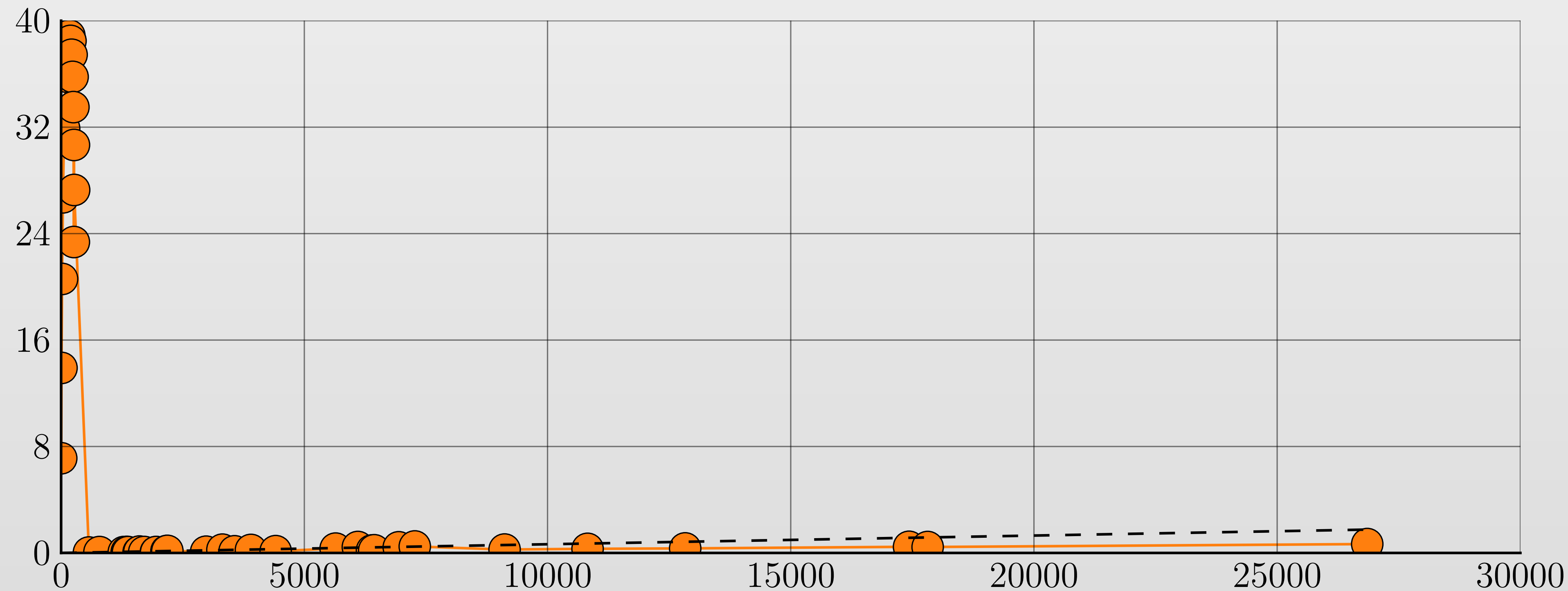
0



Ignore Hessian

$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} = \mathbf{I}$$

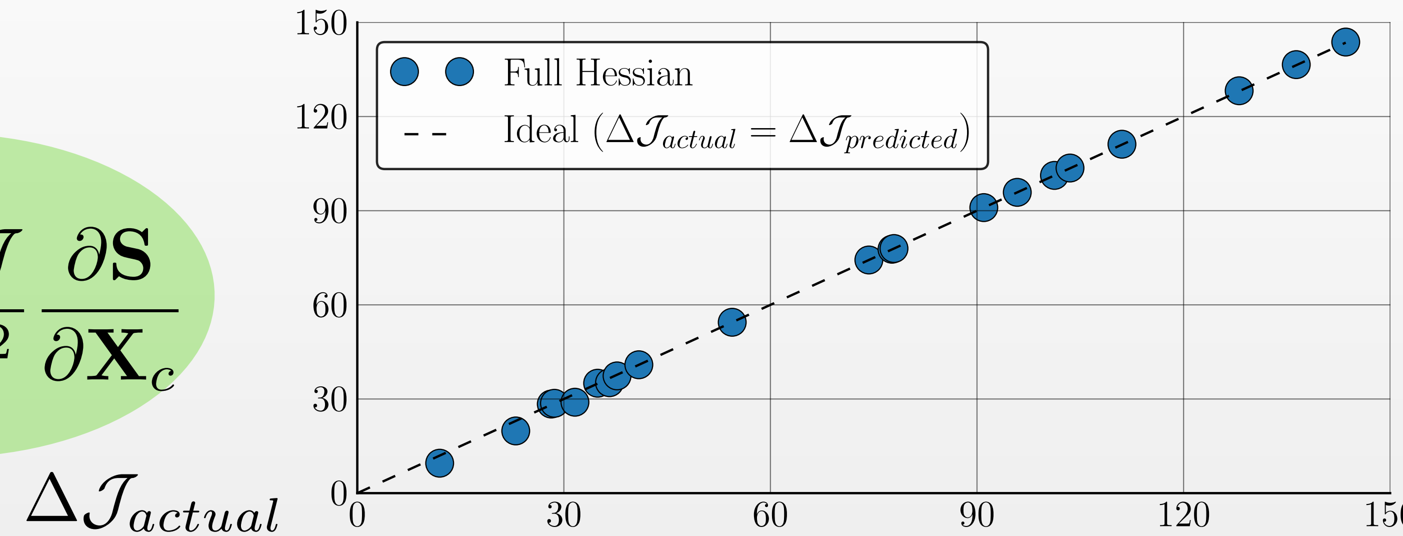




Approximations

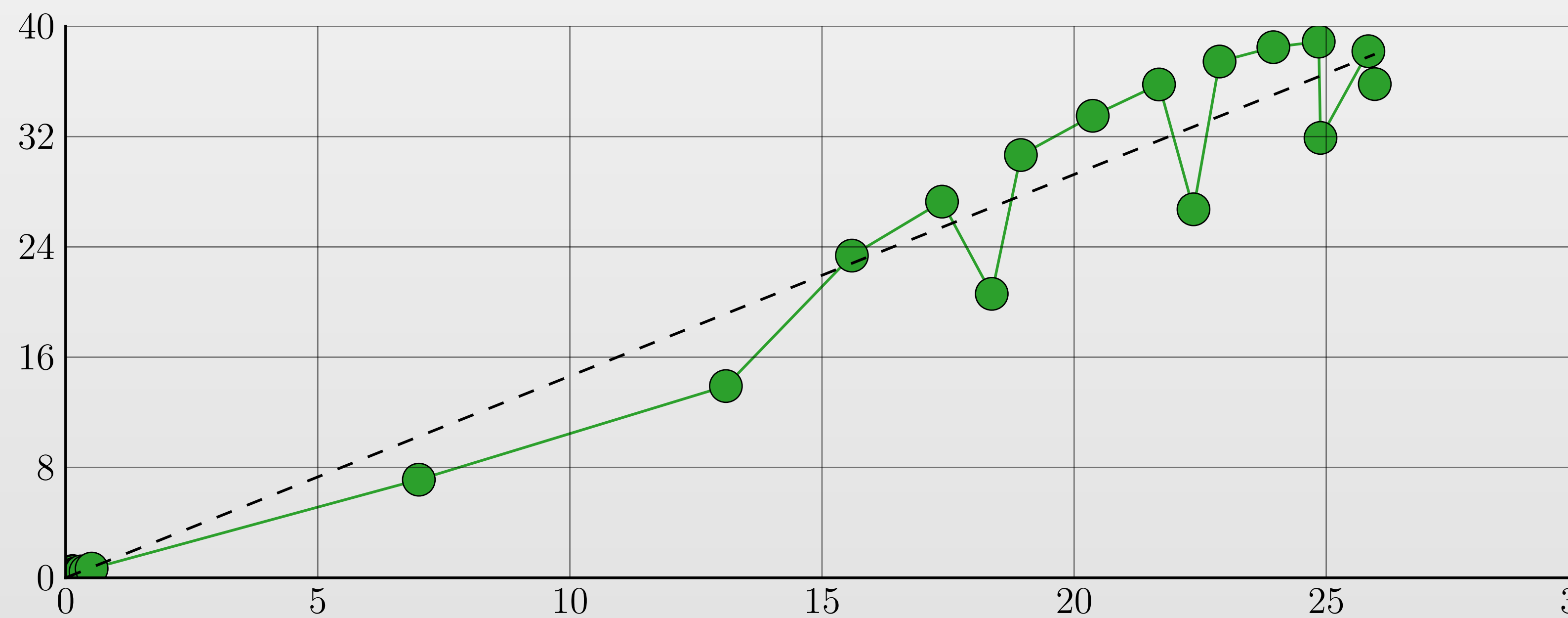
Exact Hessian

$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} = \cancelto{0}{\frac{\partial \mathcal{J}}{\partial \mathbf{S}} \frac{\partial^2 \mathbf{S}}{\partial \mathbf{X}_c^2}} + \frac{\partial \mathbf{S}^T}{\partial \mathbf{X}_c} \frac{\partial^2 \mathcal{J}}{\partial \mathbf{S}^2} \frac{\partial \mathbf{S}}{\partial \mathbf{X}_c}$$



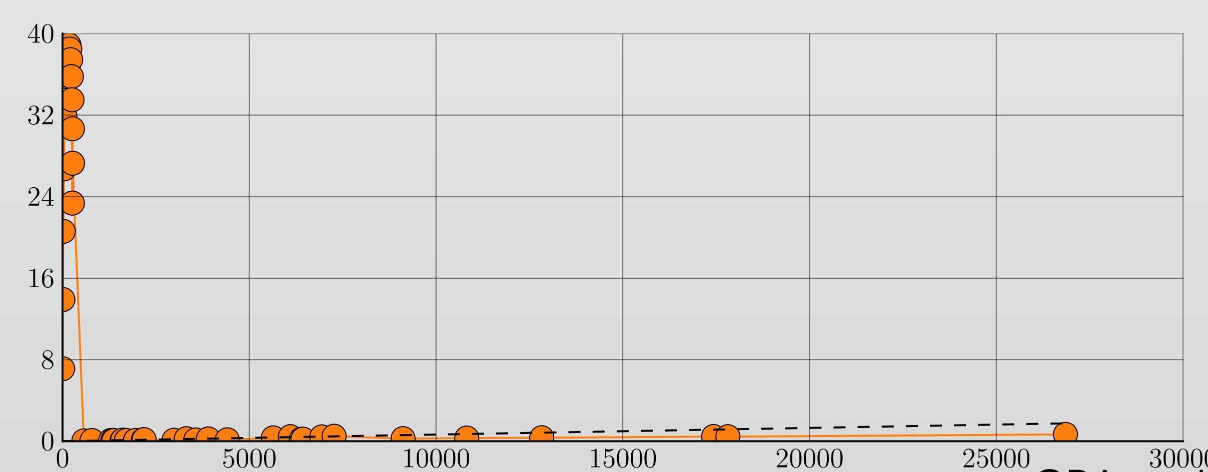
Diagonal

$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} = \cancelto{0}{\frac{\partial \mathcal{J}}{\partial \mathbf{S}} \frac{\partial^2 \mathbf{S}}{\partial \mathbf{X}_c^2}} + \mathbf{D}$$

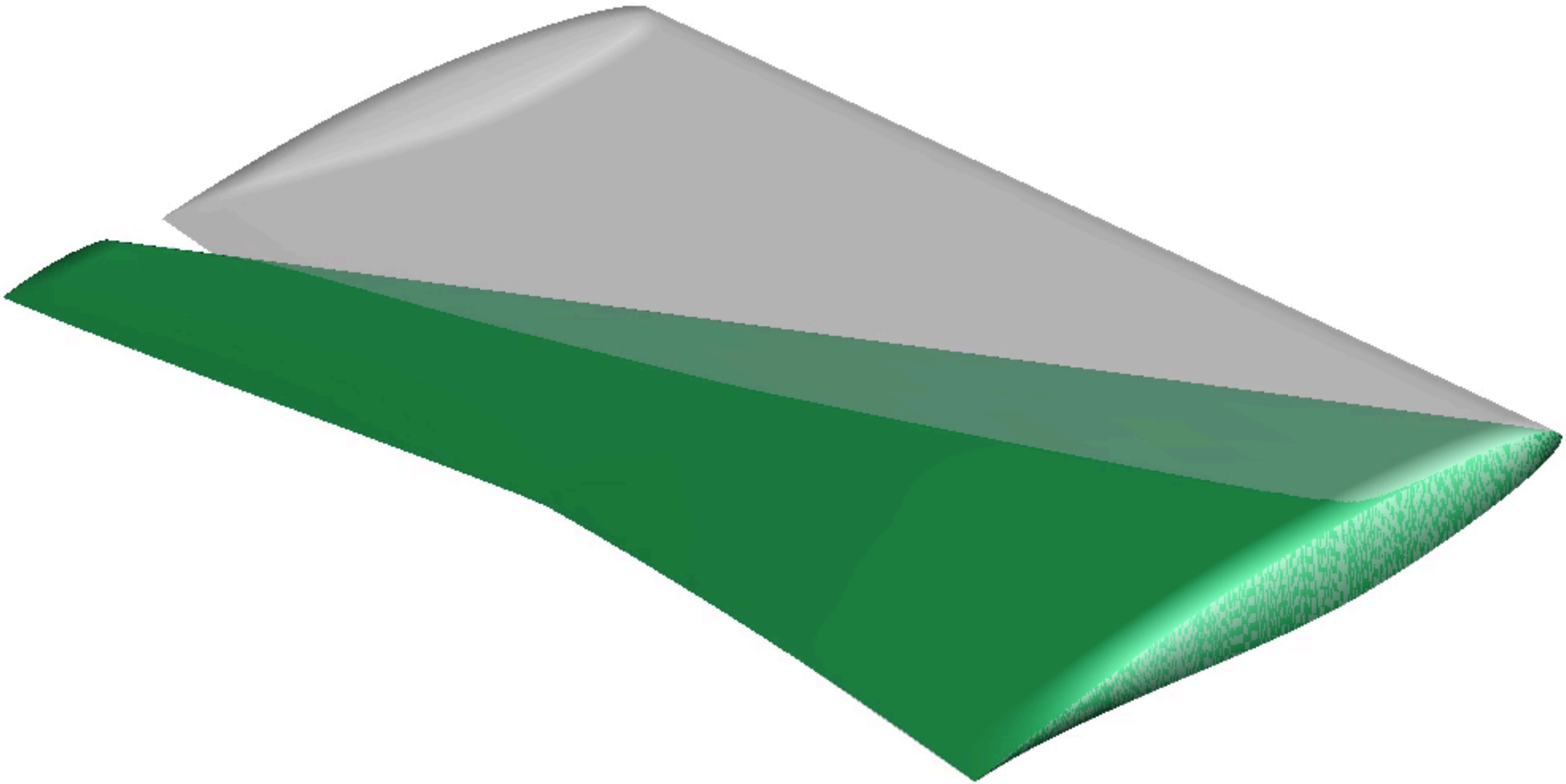


Ignore Hessian

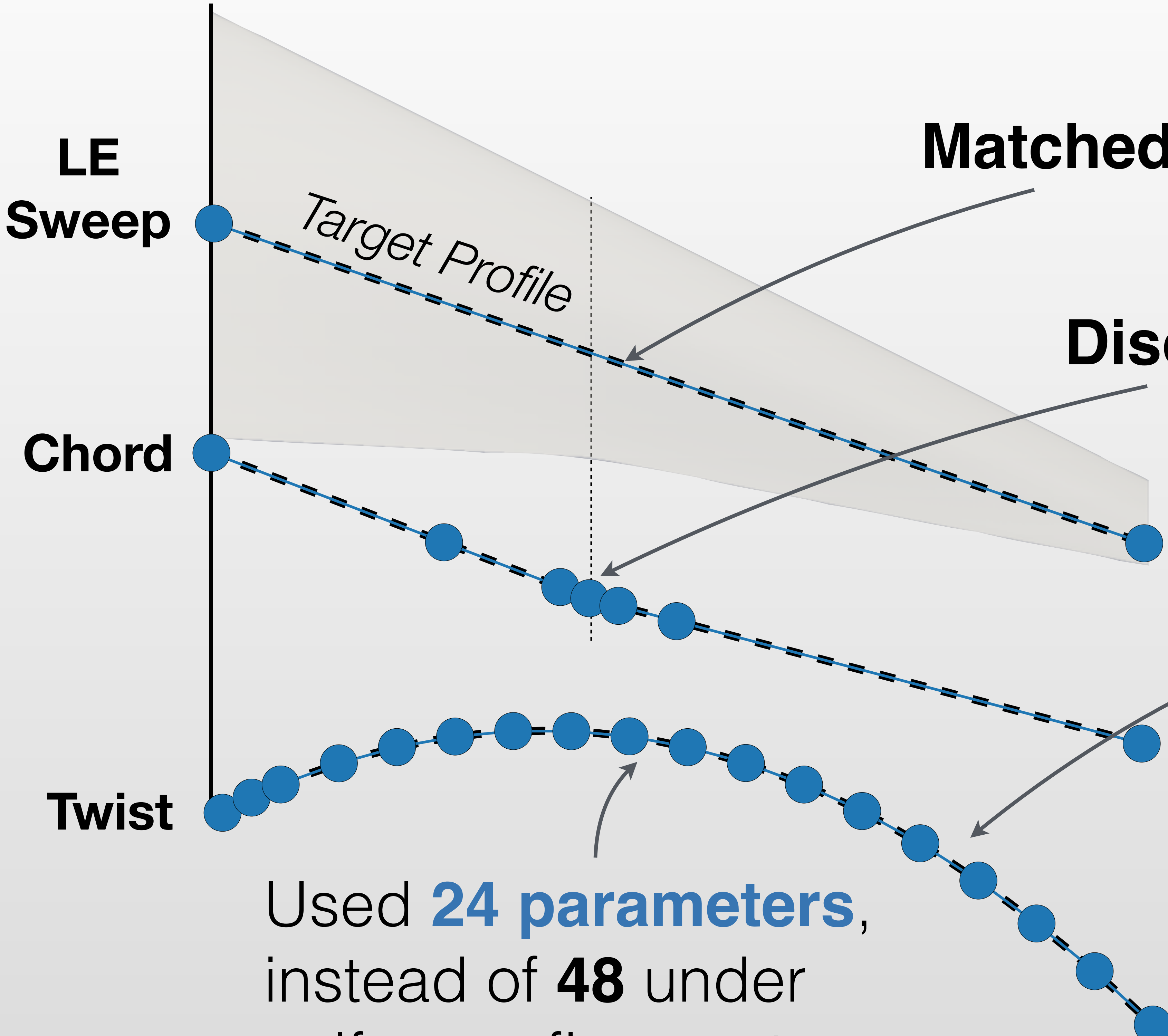
$$\frac{\partial^2 \mathcal{J}}{\partial \mathbf{X}_c^2} = \mathbf{I}$$



Shape Matching Video



Recovery of Necessary Parameters



Matched target profiles

Discovered wing break

Optimally clamping down error in twist

Used **24 parameters**, instead of **48** under uniform refinement

Adaptive Summary

- For shape-matching:
  - We can recover the necessary parameters to solve the problem.
  - Completely ignoring second-order information leads to very misleading predictions.
  - Approximation of Hessian **diagonal** is sufficient to make decent predictions.
- Ongoing work: extend results to **aerodynamic functionals.**

Conclusions

Demonstrated **adaptive shape parameterization** for aerodynamic optimization.

- **Automates** process of shape control refinement.
- Progressive, uniform shape parameterization can **accelerate optimization** (here ~3x).

Conclusions

Demonstrated **adaptive shape parameterization** for aerodynamic optimization.

- **Automates** process of shape control refinement.
- Progressive, uniform shape parameterization can **accelerate optimization** (here ~3x).

Ongoing work:

- **Adaptive** refinement can **discover** the important parameters, but second order information is essential.

Thank you!

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Adaptive Shape Control for Aerodynamic Design

NASA ARMD 2013-2015 Seedling Fund effort

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