## Shape Optimization in Adaptive Search Spaces



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## Outline

1. Introduction to Shape Parameterization
2. Automatic Adaptive Parameterization
3. Verification Studies
4. Design Examples

## Aerodynamic Shape Optimization



1. Define goals:

Minimize objective Subject to constraints

## 2. Select design variables

 (shape parameterization)
## $\min _{\mathrm{S}} \mathcal{J}(\mathrm{S})$

$\mathcal{C}_{j}(\mathrm{~S}) \leq 0$
3. Numerical optimizer iteratively modifies shape to improve performance

## Shape Parameterization

Design variable (DV) / degree of freedom
Shape parameter
Find optimal deformation
$\min _{\mathbf{X}} \mathcal{J}(\mathbf{X})$
-Shape parameterization reduces continuous design space into finite search space
-Reduces range of reachable shapes

## Static Parameterization



## Motivation

- Design of complex vehicles in unfamiliar settings, driven by highfidelity simulations.
- Choice of shape parameters impacts:
- Bias towards familiar designs.
- Ability to approximate the continuous optimal solution. (Want more DOF)
- Optimization cost. (Want fewer DOF)



## Objective

## Research Goal:

Develop system for automatic, adaptive shape parameterization refinement during optimization

Requirements:

- Gradually approach continuous optimum (convergent)
- Without a priori knowledge (automated)
- Using as few design variables as possible (adaptive)


## Previous Work

## Progressive (uniform "h"-refinement)

## Redistribution

( "r"-refinement)

- Gradually increase resolution
-(1991) Kohli and Carey — Multifidelity shape representation for structural optimization
-(1993) Marco et al. - Aerodynamic optimization with nested parameters
- Improve distribution of shape control
-(2004, 2006) Desideri and El Majd,
Duvigneau - Minimize total variation of Bezier/FFD control points
-(2012) Hwang and Martins - Equally distribute arc-length of curve between B-spline control points

These approaches are insensitive to the goals of aerodynamic optimization.

## Previous Work

Towards goal-oriented adaptation:

- (2011) Han and Zingg - Discrete refinement approach
- Restrictions: Single-component design, only localized constraints, can only add one new variable at a time
- (2014) Poole and Allen — Redistribution approach
- Restrictions: Only geometric matching of airfoils
-(2015) Anderson - Discrete adaptation approach appropriate for general aerodynamic design problems


## Contributions

- Complete system for automatic, adaptive parameterization
- Novel refinement indicator that enables adaptive parameterization for general problems:
- Multiple components
- Multiple classes of shape control
- High curvature variation in design space
- General constraints
- Several new algorithms and strategies to accelerate and automate adaptation
- First verification of robust convergence of adaptation
- Implementation, testing in a production design environment


## Outline

$\checkmark$ Introduction

- Adaptive Parameterization
- Discrete Adaptation (How?)
- Refinement Indicator (Where?)
- Adaptation Strategy

Verification
Design Examples

## Shape Control Refinement

View shape parameterization as binary tree:
Level 0

Level 1

Level 2


## Shape Control Refinement

Applicable to most parameterization techniques
Level 0
Level 1
Level 2


## Shape Control Refinement

View shape parameterization as binary tree:
Level 0

Level 1

Level 2

to two children


## Configuration Design



## Outline

$\checkmark$ Introduction

- Adaptive Parameterization
$\checkmark$ Discrete Adaptation
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## Adaptive Refinement

Goal: Determine most important candidate parameters

Add the best ones


## Previous Approach

- (2011) Han and Zingg rank parameters by magnitude of objective gradient with respect to candidate design variables. ${ }^{\dagger}$



## Prefer A, because

objective is more sensitive to it.
† (2011) X. Han, D. Zingg. "An Evolutionary Geometry Parametrization for Aerodynamic

## Limitations of Previous Approach

- Ignores constraints

Inconsistent units
Ignores curvature variation Insensitive to redundancy

Drag is more sensitive to A , but thickness constraint would be violated

B offers more real potential, despite lower objective gradient


## Limitations of Previous Approach

Ignores constraints

- Inconsistent units

Ignores curvature variation Insensitive to redundancy


## Limitations of Previous Approach

Ignores constraints
Inconsistent units
Ignores curvature variation $\frac{\partial^{2} \mathcal{J}}{\partial X_{c}^{2}}$
Insensitive to redundancy
Insensitive to redundancy


## Limitations of Previous Approach

Ignores constraints
Inconsistent units
Ignores curvature variation

- Insensitive to redundancy

Either one would be useful, but not both


## New Refinement Indicator



## Expected Feasible Design Improvement

## KKT system

## $\longrightarrow$ Gradients of active constraints

Hessian $\longleftarrow\left[\begin{array}{c}\mathcal{H} \\ \left(\frac{\partial \mathcal{C}^{a}}{\partial S}\right)^{\top}\end{array}\right.$

## Refinement Indicator

$$
\Delta \mathcal{J}_{\text {exp }}^{\infty}=\frac{1}{2}\left\langle\left(\frac{\partial \mathcal{J}}{\partial \mathrm{~S}}+\lambda \frac{\partial \mathcal{C}^{a}}{\partial \mathrm{~S}}\right), \mathcal{H}^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathrm{~S}}+\lambda \frac{\partial \mathcal{C}^{a}}{\partial \mathrm{~S}}\right)\right\rangle
$$

Expected feasible objective reduction in candidate search space:

KKT stationarity
0 at optimum

$$
I \equiv \Delta \mathcal{J}_{\text {exp }}^{\infty}=\frac{1}{2}\left\langle\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\boldsymbol{\lambda} \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{c}}\right),(\mathcal{M H})^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\boldsymbol{\lambda} \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{\mathbf{c}}}\right)\right\rangle
$$

Use as refinement indicator


Has sensible units

$$
\begin{gathered}
{[I]=\frac{\text { Drag }}{\mathrm{ft}}\left(\frac{\mathrm{ft}^{2}}{\mathrm{Drag}}\right) \frac{\text { Drag }}{\mathrm{ft}}=\text { Drag }} \\
\text { "expected drag reduction" }
\end{gathered}
$$

## Refinement Indicator

$$
I=\frac{1}{2}\left\langle\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\boldsymbol{\lambda} \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{c}}\right),(\mathcal{M H})^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\boldsymbol{\lambda} \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{\mathbf{c}}}\right)\right\rangle
$$

Explicitly accounts for constraints

Hessian matrix


## Indicator Computation - Gradients



## Indicator Computation - Hessian Estimation

$I=\frac{1}{2}\left\langle\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\lambda \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{c}}\right),(\mathcal{M} \overline{\mathcal{H}})^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\lambda \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{\mathrm{c}}}\right)\right\rangle$
Estimate Hessian from quasi-Newton approximation in previous space

BFGS

$$
\left[\begin{array}{ll}
\mathbf{B}_{0}^{k}=\mathbf{I} & \mathbf{H}_{0}^{k+1} \approx \mathcal{P}\left(\mathbf{B}_{n}^{k}\right) \\
& \\
\mathbf{B}_{n}^{k} \approx \mathbf{H}_{n}^{k} & \begin{array}{l}
\text { Prolong to } \\
\text { finer space } \\
\text { (see dissertation) }
\end{array}
\end{array}\right.
$$



Hessian Diagonal

## Outline

$\checkmark$ Introduction

- Theory and Approach
$\checkmark$ Discrete Adaptation
$\checkmark$ Refinement Indicator
- Adaptation Strategy

Verification
Design Examples

## When to refine?



## Growth rate



## Adding Multiple Parameters

- Adaptation: "Find the best $N$ out of $M$ parameters"
- Properly a combinatorial optimization problem
- Not separable for most deformers
- But conducive to approximate solutions
- I use an approximate constructive (greedy) algorithmt

${ }^{\dagger}$ (2015) Anderson, G.R., Aftosmis, M. J. "Adaptive Shape Control for Aerodynamic Design." AIAA 2015-0398


## Regularity

Require regularity in


## Outline

$\checkmark$ Introduction
$\checkmark$ Theory and Approach

- Verification
- Correctness - Does the indicator predict actual design improvement?
- Robustness - Does the approach always converge to the continuous optimum?

Design Examples

## Verification Study 1: Geometric Shape Matching



## Initial Parameterization



## Shape Matching under Initial Parameterization



## Indicator Validation



## Indicator Validation

For each candidate:

1. Predict design improvement. With indicator:

$$
I=\frac{1}{2}\left\langle\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\lambda \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{c}}\right),(\mathcal{M H})^{-1}\left(\frac{\partial \mathcal{J}}{\partial \mathbf{X}_{c}}+\lambda \frac{\partial \mathcal{C}^{a}}{\partial \mathbf{X}_{\mathbf{c}}}\right)\right\rangle
$$

2. Measure actual improvement.



Search Direction

## Indicator Validation

Correlate predicted and actual design improvement


## Approximations




## Excellent prediction

## Acceptable

 some redundancy

Identity



## Poor ranking

systematic difference
between classes of shape control

## Recovery of Necessary Parameters

|  | Matched target profiles |
| :--- | :--- |

Sweep Targot Drofile Discovered wing break

Twist


## Balanced adaptation

 of three different classes of shape control
## Verification Study 2: Pressure Signature Matching

## Objective: Match target pressure profile <br> $\mathcal{J}=\frac{1}{2} \sum_{i=1}^{N_{\text {verts }}}\left(p_{i}-p_{i}^{*}\right)^{2}$

## Parameterization:

2D Radial basis functions (localized bumps)
Flow Solver: Cart3D Optimizer: SNOPT


## Video - Results



## Convergence to Continuous Optimum



## Convergence to Continuous Optimum



## Convergence Rate

## Efficient in use of design variables

Asymptotic convergence rate of $\mathcal{J}_{\star}^{k}-\mathcal{J}_{\star}^{\infty}$

|  | Uniform | Adaptive |  |
| :---: | :---: | :---: | :---: |
| Case |  | Strategy 1 | Strategy 2 |
| 1 | 2.6 | 8.3 | 5.0 |
| 2 | 2.4 | 5.2 | 5.6 |
| 3 | 2.7 | 5.7 | 4.7 |
| mean | $\mathbf{2 . 6}$ | $\mathbf{5 . 7 5}$ |  |
| $\frac{\Delta \mathcal{J}}{\Delta N_{D V}} *$ | $\sim \mathbf{6} \times$ | $\sim \mathbf{5 4 \times}$ |  |

* Reduction in objective for $2 \times$ increase in $N_{D V}$


## Refinement Patterns

Automatic shape control clustering at leading and trailing edges


Different adaptation strategies result in similar patterns

## Adaptive System

Naive initial parameterizations

Wilbur

- Continuous optimum
- Adapted parameterization



## Outline

$\checkmark$ Introduction
$\checkmark$ Theory and Approach
$\checkmark$ Verification

- Design Examples
- Implementation
- Sonic boom signature matching
- Adaptive flaps for Truss-braced wing


## Discrete Geometry

- Direct manipulation of surface tessellations
- CFD-ready - always high resolution
- Allows optimization of "legacy" geometries


## O blender

- Serves as geometry engine for optimization
- Script-driven surface mesh deformation
- Implemented a number of custom deformation techniques
- (2012) Anderson and Aftosmis, "Parametric Deformation of Discrete Geometry for Aerodynamic Shape Design". AIAA Paper 2012-0965.


## Cart3D

- Cartesian cut-cell method with automated meshing of complex configurations
- Inviscid solver with adjoint-driven
- Adaptive meshing
- Error estimates
- Functional gradients



Output-adaptive meshing

## Optimizer

SNOPT - Sparse Nonlinear Optimizer

- Quasi-Newton method - gradually builds up Hessian approximation
- SQP method - handles nonlinear inequality constraints



## Boom Design



## Inverse Design Procedure



## Seeb-ALR

## Target

Seeb-ALR


## Target Nearfield Signature



## Baseline Geometry

Mach 1.6

## Baseline

Cone
$\alpha=0$


## Mesh Adaptation



## Mesh Adaptation



## Adaptive Parameterization



## Adaptive Parameterization



Level 1
Level 2

## Video - Results



Objective Convergence


## Adaptive Wing Morphing


(2015) Rodriguez, Aftosmis, Nemec, Anderson, "Optimized off-design performance of flexible wings with continuous trailing-edge flaps." AIAA Paper 2015-1409, AIAA SciTech 2015, Kissimmee, FL.

## Truss-Braced Wing (TBW)



## Flap Adaptation Procedure

( 1) Morph: Optimize flap deflections for minimum drag.
2) Refine flap topology: Add the one* additional flap that would best allow the drag to be reduced.

$$
\begin{aligned}
\mathcal{J} & =C_{D} \\
M & =0.7 \\
C_{L} & =0.766
\end{aligned}
$$

*Add flaps one at a time, because the cost associated with every flap is real - want to find minimal parameterization!


## Flap Refinement

## Strut

Jury strut

Initially two monolithic flaps, can be subdivided...
span-wise
or stream-wise

uniformly
or adaptively

## First Step



## 4 ways to split

## Priority Queues




## Verification of Ranking



## Flap Deflection History

## Final flap topology




Baseline geometry has substantial wave drag through truss

## Cost vs. Flap Count



## Conclusions

- Demonstrated adaptive shape parameterization system for automated, high-fidelity aerodynamic optimization.
- Enables hands-off design exploration for unfamiliar problems.
- Provides feedback about the design problem.
- Verification studies confirm that robust convergence to continuous optimum is possible.
- A careful adaptive strategy makes the approach substantially more efficient both in terms of design variables and computational time.


## New Techniques

- Goal-oriented refinement indicator targeting high potential shape parameters.
- Substantially improves results over previous best indicator, appropriate for general classes of problems.
- Leverages information already available during optimization no a priori knowledge required.
- Approximate Hessian estimation (prolongation operator)
- Could also be used to accelerate design in finer design spaces.
- Constructive algorithm to efficiently find an approximate solution to the combinatorial adaptation problem.
- Cost-benefit approaches to automatically determine how many parameters to add and when to trigger refinement.


## Optimization Benchmarks

Transonic wing and airfoil design benchmarks


- Combined two automated, adaptive elements:



Progressive parameterization

Adaptive mesh refinement
${ }^{\dagger}$ (2015) Anderson, Nemec, Aftosmis. "Aerodynamic Shape Optimization

## Publications

1. Anderson and Aftosmis, "Parametric Deformation of Discrete Geometry for Aerodynamic Shape Design". AIAA Paper 2012-0965, 50th AIAA ASM Meeting and Exhibit, Nashville, TN, January 2012.
2. Anderson, Aftosmis, Nemec, "Constraint-based Shape Parameterization for Aerodynamic Design". ICCFD7 Paper-2001. Seventh International Conference on Computational Fluid Dynamics (ICCFD7), Big Island, HI, July 2012.
3. Anderson and Aftosmis, "Adaptive shape parameterization for aerodynamic design." NASA Technical Memorandum, May 2015.
4. Rodriguez, Aftosmis, Nemec, Anderson, "Optimized off-design performance of flexible wings with continuous trailing-edge flaps." AIAA Paper 2015-1409, AIAA SciTech 2015, Kissimmee, FL, http://dx.doi.org/10.2514/6.2014-1409, January 2015.
5. Anderson, Nemec, Aftosmis, "Aerodynamic shape optimization benchmarks with error control and automatic parameterization." AIAA Paper 2015-1719, Kissimmee, FL, http://dx.doi.org/ 10.2514/6.2015-1719, January 2015.
6. Anderson and Aftosmis, "Adaptive shape control for aerodynamic design." AIAA Paper 2015-0398, AIAA SciTech 2015, Kissimmee, FL, http://dx.doi.org/10.2514/6.2015-0398, January 2015.

## Future Work

Major outstanding topic:

- Discovering effective classes of shape control



## Backup Slides

## But How Fast Is It?



## Low resolution: <br> Faster design improvement

High resolution:
Better designs


Wall clock time
In minutes, plotted at major search iterations, on 20 lvybridge cores

## Progressive vs. Static



Fast improvement in coarse search spaces, but ultimately approaching full design space.


Wall clock time
In minutes, plotted at major search iterations, on 20 lvybridge cores

## Cost

## Factors contributing to acceleration:

- Early on there are few design variables:
- Accelerates BFGS rate of improvement w.r.t search direction.
- Reduces \# of shape sensitivities and gradient projections.
- Later, more design variables are added, preventing optimization from stalling.



## Wall clock time

In minutes, plotted at major search iterations, on 20 lvybridge cores

## Impact of Parameterization



## Progressive vs. Static



## Adaptive vs. Uniform



## Goals of Adaptation



